Wingate's RULE OF PROPORTION, IN ARITHMETICK AND GEOMETRY: OR, GUNTER'S LINE

Newly rectified by Mr. *Brown* and Mr. *Atkinson*, Teachers of the Mathematicks.

Fitted for all Artists for Measuring and Building.

Whereinto is now also inserted the Construction of the same Rule, and a farther Use thereof, in Questions that concern

Astronomy,	Gaging of Vessel,
Dialling,	Military Orders,
Geography,	Interest and
Navigation,	Annuities.

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on, Printed by *R. H.* for *W. Fisher*, *T. Passinger*, *R. Boulter*, and *R. Smith*, 1683.

TO MY

Worthy Friend,

AND ABLE

MATHEMATICIAN,

Mr. *John Collins*

OF

LONDON.

SIR,

NOt long after my Arrival in this City, having divulged the Instrument (whose Uses I explain in this little Treatise) and discoursed of some of the conveniences thereof, I was given to understand by

divers

divers, that if pains were bestowed upon that Subject, the Labour therein taken might obtain good Reception: This (to say truth) hath given me Encouragement thereof to say somewhat, and (having caused it to see the Light) to shelter it under your Protection: Neverthelesse you shall pardon me, for that by presuming to procure unto it from thence Credit and Recommendation I have expressed a willingnesse to testifie, how much I am,

Your Servant,

Edmond Wingate.

THE PREFACE TO THIS TRANSLATION.

AMongst the many rare Effects produced by the noble Invention of *Logarithmes*, the projection of the *Rule* of *Proportion* is not the least, which being first discovered by that Learned and Industrious Artist *Edm. Gunter* (late Professor of *Astronomy* in *Gresham* Colledge, *London*, deceased) was by me (in *Anno* 1624.) transported into *France*, and there communicated to most of the chief-

eſt Mathematicians then reſiding in *Paris*, who apprehending the great benefit that might accrue thereby, importuned me to expreſs the uſe thereof in the *French* Tongue; which being performed accordingly, I was adviſed by Mr. *Alleaume* (the King's Chief Ingenier) to dedicate my Book to *Monſieur*, the then King's only Brother, now Duke of *Orleance*: Nevertheleſs this Work (as it was there publiſhed) coming forth as an *Abortive*, the publiſhing thereof being ſomewhat haſtned, by reaſon an Advocate of *Dijon* in *Burgundy* began to print ſome Uſes thereof, which I had in a friendly way communicated unto him) I thought it not worthy to ſee the Light here in *England*, eſpecially in regard Mr. *Gunter* himſelf had learnedly explained the uſe thereof in a far larger Volume: Howbeit having now of late by reaſon of the preſent Troubles) had too much leiſure

from

from my other Employments and Calling, to look back to those Studies, wherewith in my younger time I used to busie my self; And having also upon that occasion bethought my self, how divers necessary Additions might be fitly inserted into that Work, and many inconveniences in the use of that *Instrument*, which before did usually incumber the Practitioner, might be removed; I have adventured to let this Translation appear; In which you shall find expressed, as succinctly and plainly as I could, the use of that *Rule* in the form as you find it annexed to this Book; Not that I would confine any man to use such a form and no other, but because the Operations are thereupon understood and performed more perspicuously and plainly, then (as I conceive) they would be, if the Lines were thereupon otherwise described; Howbeit the use thereof

of being in this form once gained, the Practitioner may then uſe that way of deſcribing it, which ſorts beſt with his own humor.

Having thus acquainted you with the occaſion of publiſhing this Treatiſe, leſt I may now expoſe it to prejudice, give me leave to premiſe theſe few Advertiſements following: Firſt, therefore, it is deſired, that he, who intends to read this Book with profit, ſhould have a proper *Genius* and Phanſie for the *Mathematicks*, not only ready to conceive *Mathematical* Notions; but likewiſe able to wreſtle with them, and apt to take pleaſure in them: For, *De quolibet ligno non fit Mercurius*. Again, it is expected he ſhould be aforehand furniſhed with competent knowledge in thoſe Sciences, *viz.* 1. In *Arithmetick* he ought to be acquainted with the Nature of Numbers, whole and broken, abſolute and relative; with Numera-

tion,

tion, Addition, Subſtraction, Multiplication, Diviſion, the Rule of Three, direct and inverſe; with the Nature and Extraction of Roots, Square and Cube; And with the right uſe of *Logarithms*: 2. In *Geometry*, to be verſt in the Doctrine of Triangles, plain and ſpherical, and (in ſome competent meaſure) to know their nature, together with the way and reaſon of their dimenſion; As alſo the dimenſion of other Geometrical Figures: 3. In *Aſtronomy*, *Dialling*, and *Geography*, to underſtand that the Problems which concern them, are reſolved by the particular application of the Doctrine of Spherical Triangles to thoſe ſeveral Sciences: 4. In *Navigation*, to be indifferently well read in ſuch Authors as have explained that Art, and to be able therein alſo to make uſe of the Doctrine of Triangles: With the knowledge of theſe things (I ſay) and

and the like he ought to be (in ſome reaſonable ſort) ſupplied, that intends to make a right and complete uſe of this Treatiſe: For, none (I preſume) will expect to find an intire Body of the *Mathematicks* in this ſmall Bulk, which is only intended for an *Enchiridion* or Manuel of ſuch Mathematical Rules and *Analogies*, as may moſt properly ſerve for the reſolution of Problems, which may be wrought upon this *Inſtrument*: And therefore I wholly refer the *Reader* for demonſtrations and larger explanations of the matters in this Book contained, to the further ſcrutiny of other Authors; Not doubting but that (upon due peruſal hereof) he will find as much inſerted, as ſhall be thought neceſſary to diſcover the manifold and exquiſite uſe of the ſame *Inſtrument*. But here I would not be miſtaken, as if I did totally exclude all others, who are

not

not prepared with ſuch an Univer-ſal Knowledge in the *Mathematicks*, from having any capacity at all of underſtanding this Book; For, if he be only in part acquainted with ſome of the abovementioned Learning, he may be able to make uſe of this *Inſtrument* according to that degree of Knowledge which he hath therein; For Example, if he only know *Multiplication* and *Diviſion*, this Treatiſe will inſtruct him how to *multiply* and *divide* upon the *Rule*, and ſo in like ſort of the reſt: Howbeit (as I ſaid before) if he intend to have an intire underſtanding of the uſes of this *Inſtrument*, he muſt be alſo furniſhed with an intire knowledge of all the *Mathematicks*; becauſe it is ſubſervient to every Branch of thoſe Sciences: And then the conveniency thereof will have ſuch Latitude, that it will not be confined to thoſe uſes only promiſed in the Title of this Book, but

likewiſe

likewiſe (by the variety of Rules and Examples therein found) may be readily and fitly applied to other Arts and Profeſſions not there remembred; As namely, in *Fortification*, the Ingenier may here be taught how to find the Sides of his *Polygonical* Figures, the *Lines* of Fortification according to the Rules of that Art, the *quantity* of Trenches and Ramparts, how to order and eſtimate the labour and work of Pioners, and the like. The *Surveyor* alſo may here furniſh himſelf with divers expeditious diſpatches, for the taking of diſtances, the ſumming up of Plots, being firſt divided into Triangles, the diſtribution of Fields or Lordſhips to ſeveral Perſons, the cutting off any part of a Triangle or Plot according to any quantity propounded, *&c.* The like may be ſaid of *Muſick*, *Architecture*, the *Proſpectives*, *Gunnery*, *&c.* The *Goldſmith* alſo, and *Mint-Maſter*

ſter may here learn how to temper their Allegations: The *Merchant* and *Tradeſman*, how to reſolve queſtions of Partnerſhip, and to caſt up the value of their Commodities: The *Juſtice* of *Peace* and *High Conſtable*, how to rate a Town, Hundred, or County, *&c.* All which and much more muſt be wholly left to the diſcretion of thoſe, that will take the pains to underſtand the uſe of the ſaid *Inſtrument*; which (I perſwade my ſelf) no man (affecting the *Mathematicks*) will think much to undergo, conſidering the benefit he may reap thereby, and the delight he may take therein; For, by help thereof, and of a pair of Compaſſes, only ſix Inches long, he may reſolve with requiſite exactneſs any Propoſition in the Arts and Sciences above remembred (which comes within the bounds of ordinary practice) without the help of Pen or Paper, and ſhall thereby alſo

perform

perform more in one hour, then otherwife (I mean by ordinary *Arithmetick*) he fhall be able to difpatch in two whole days.

But it may be objected, if this *Inftrument* be of fuch excellent ufe as is here pretended, why hath it not been heretofore of greater efteem, it being now above twenty years fince it was firft invented? This Objection may be anfwered divers ways: 1. It is no eafie matter to drive men out of their old track, efpecially when they have entertained an opinion that there can be none better. 2. Again, the ufe thereof in the point of *Numbring* upon the *Rule* (which ought to be accounted the chiefeft, and indeed the ground of all the reft) hath not been heretofore (under favour) fo fully explained, as here you fhall find it: For, albeit (I confefs) it were great prefumption in me to affume to my felf the reputation of

having

having better abilities to describe any of the uses thereof, then Mr. *Gunter* himself had, who first invented it; yet this I can aver upon mine own knowledge, that he did forbear to explain the use thereof, because he took it for granted none would meddle with it but such only as were already well able to understand how to number upon it, having before-hand acquainted themselves with the manner of *Numbring* upon *Scales*, and with the nature of *Logarithms*: For, when after my return out of *France*, I importuned him to make a fuller explanation, how to number upon it, to the end the use thereof might by that means be made more publick, his answer was, *That it could not be expected the Rule should speak*; Intimating thereby, that the Practitioner should (in that point) rely much upon discretion, and not altogether depend upon Precepts and Examples. But last-

ly,

ly, the chiefeſt cauſes why this *Inſtrument* hath been hitherto obſcured and the uſes thereof no better known to the World, are theſe. 1. The Difficulty of deſcribing the Lines thereupon with convenient exactneſs: 2. The trouble of working thereupon by reaſon (ſometimes) of too large an extent of the Compaſſes: 3. The importableneſs thereof, it being requiſite for working upon ſuch a *Rule* (only two foot long) to uſe a pair of Compaſſes of nine Inches: 4. The charge of purchaſing ſuch an *Inſtrument* made of Braſs or Wood; For, none but ſuch have been heretofore uſed. For remedy of the firſt of theſe, I have cauſed the Plate, whereupon this *Inſtrument* is Printed, to be protracted with a great deal of care and circumſpection, ſo that I dare affirm it to be as exactly drawn (for the main and moſt conſiderable Diviſions thereof) as may be expect-

cd

ed from Art: For the ſecond, having there three ſeveral Lines of *Numbers* by degrees one leſs than another, when the Compaſſes are too little for one, you may uſe another, alſo *Croß-work* upon the greateſt Line will prevent the too great extenſion of the Compaſſes; ſo that it will be requiſite to uſe with this *Inſtrument* (as it is now contrived) a pair of Compaſſes only ſix Inches long, as I ſaid before; and yet the Diviſions of this (I mean upon the great Line of *Numbers*) are near as large again, as thoſe upon Mr. *Gunter*'s Rule of the like length: The third and fourth impediments may alſo be remedied, if in ſtead of Braſs or Wood you uſe the impreſſion of the ſaid Plate upon Vellum or Imperial Paper, which may either be rolled up and couched in a little Box, or otherwiſe paſted upon a Ruler, either flat, to uſe at home, or round, to be carried

in

in a hollow Staff or Cane together with the Compaſſes, which are to be uſed therewith. Alſo divers uſeful conveniences ſhall you meet withall in this Edition of the *Rule*; as namely, a readier way of finding out *Mean-Proportions*, the *Extraction* of Roots by Inſpection only, without aid of Pen or Compaſſes, and the like: For further diſcovery of all which I refer you to the Book it ſelf, hoping that my real intention to advance the Publick Good will procure from the Ingenuous *Reader* a favourable conſtruction of what he ſhall therein find not wilfully miſtaken.

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By *James Atkinson*.

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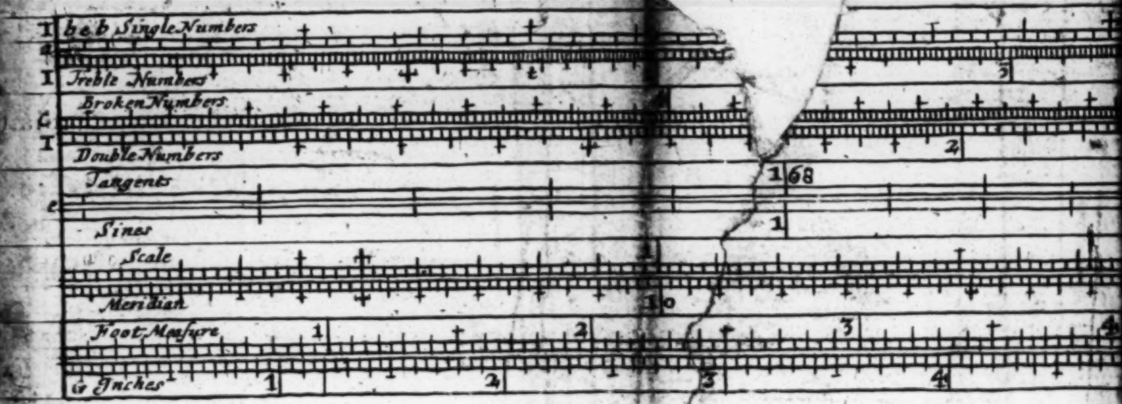
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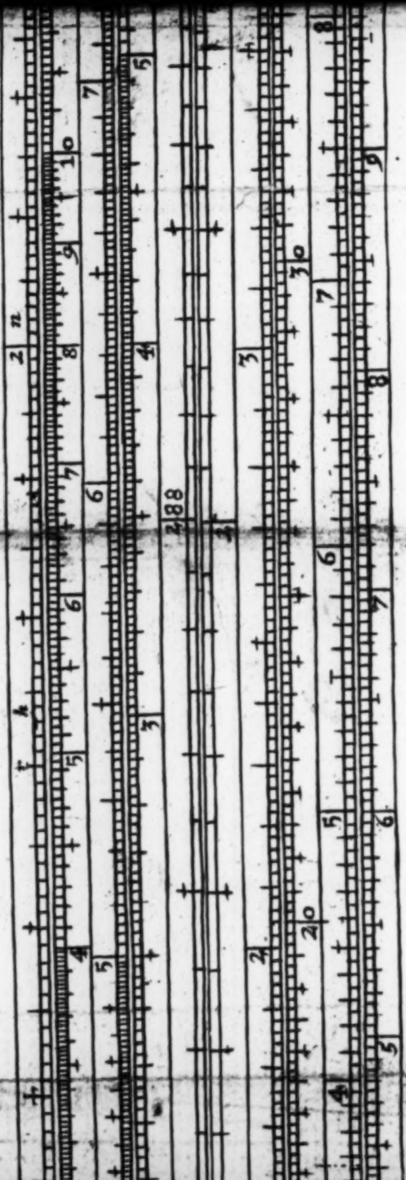

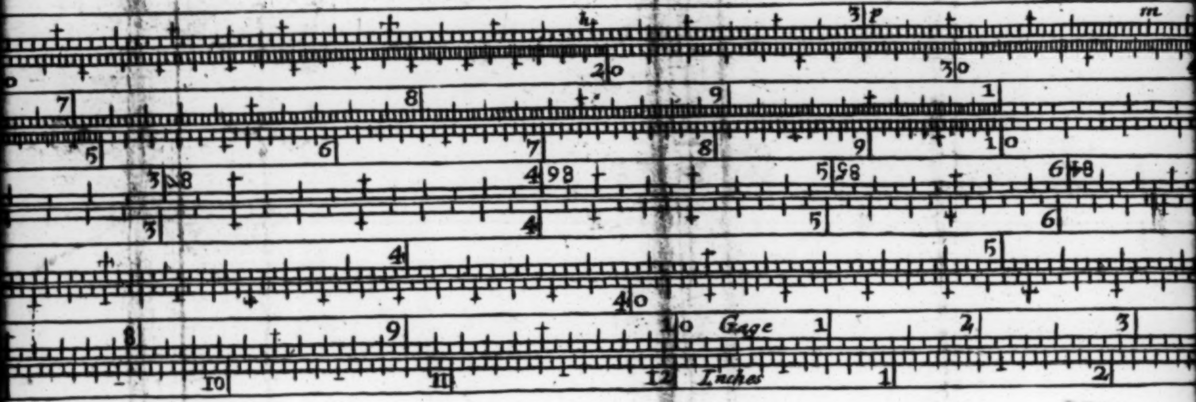
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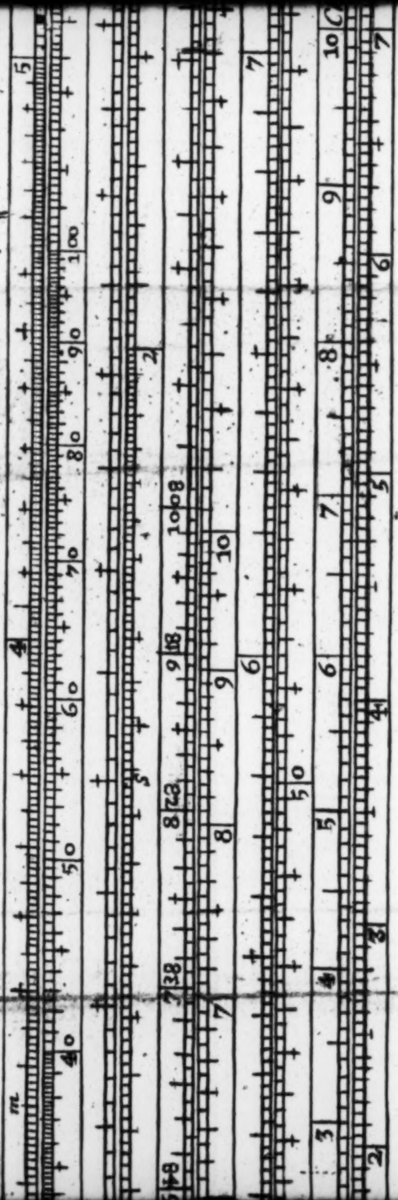

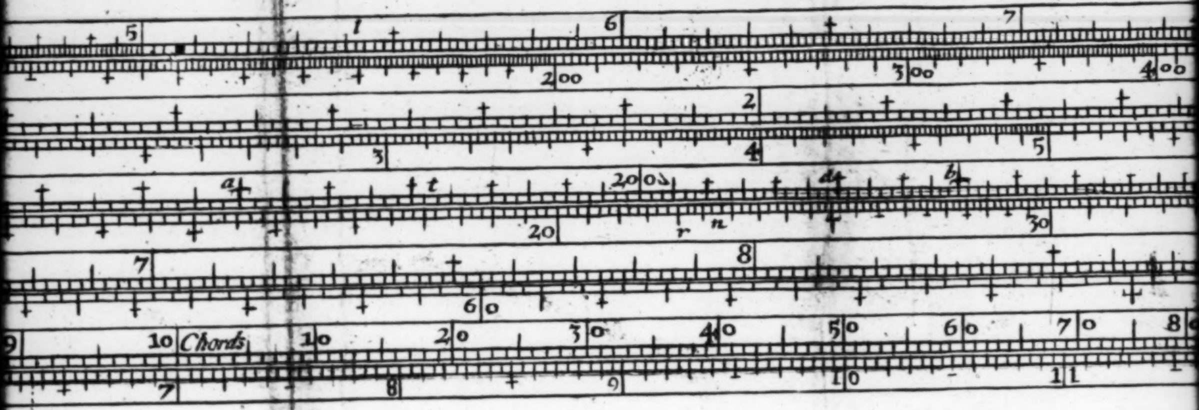
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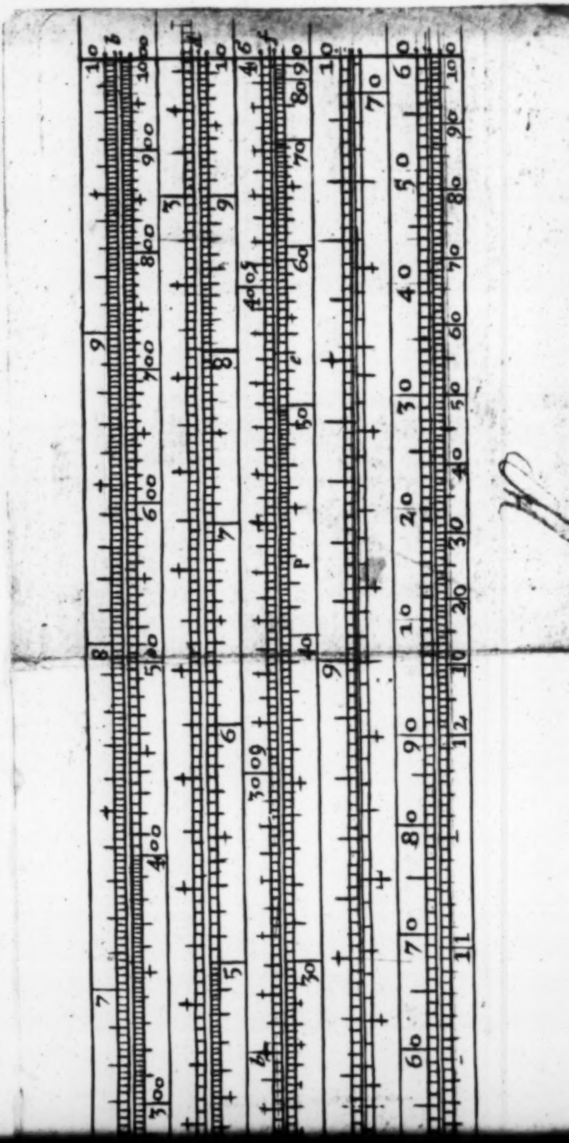

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THE USE OF THE Rule of Proportion in Arithmetique and Geometrie.

CAP. I.

The Description of the Scales projected upon the Rule of Proportion.

UPon the five *Lines of the Rule of proportion, there are ten several Scales projected*, viz. *two upon each common or middle Line, the one having the Divisions thereof shooting downwards, the other* [illegible]. So the first two Scales meet upon the middle or common Line *a*, *b*, the next two upon the Line *c*. *d*, &c.

The uppermost or first Scale of the Rule is a single Line of Numbers; first divided into nine unequal parts, called *Primes*, and distinguished by the Figures, 1.2.3.4.5. 6. 7. 8. 9. And then, each of those *Primes*, subdivided into ten other Parts (according to the same Reason) called *Tenths:* And again, each of those *Tenths* subdivided, or at least supposed to be subdivided into ten other Parts, as the length of

the Rule will admit: For *Example*, upon the Scheme of our Rule (hereunto annexed) which is supposed to be about two foot and three inches long between the end-lines in the four first *Primes* (*viz.* between the Figures 1 and 5) each *Tenth* is really subdivided into ten Parts; but in the rest of the *Primes* (*viz.* between the Figure 5, and the end of that Scale) each *Tenth* is divided but into five Parts; and therefore each of those five Parts ought to be esteemed to have the value of 2; and the said tenth parts of those *Tenths* are hereafter called *Centesmes*: Lastly, each of those *Centesmes* is also supposed to be subdivided into ten lesser Parts, which are hereafter called *Millains*: By all which you may observe, that the longer the Rule is, the more small Divisions it will admit, and the shorter it is, the fewer.

The second Scale is another Line of Numbers thrice repeated: This Scale shoots upwards upon the Common Line *a*, *b*, and being of a lesser Volume than the former, must in some Parts thereof content it self with less Divisions, *viz.* from the Figure of 5 to the end of that Scale the *Tenths* are only divided into two Parts, and therefore each of those two Parts ought to retain the value of five: All the three Parts of this Scale being taken together, are hereafter (for distinction sake) called the *Little Line of Numbrs*, and are in their use distinguished by the first, second, and third Part, as they lie in order; They are also of singular use for the ready discovery of the Cube-root, and for the resolution of other necessary operations, as shall be shewed hereafter.

The third Scale is the first Scale repeated, taking his beginning from the middle of the Rule, and being broken off at the upper end thereof, is afterwards continued from the lower end of the same to the place where it first began. This Scale abuts downwards upon the Common Line *c*, *d*; and the first and this being taken together are hereafter called the *Great Line of Numbers*, whereof

whereof the first Scale is called the first Part, and this the ſecond.

The fourth Scale is another Line of Numbers twice repeated: This Scale ſhoots upwards upon the Common Line *c, d,* and being intirely taken together, is hereafter called the *Mean Line of Numbers:* It conſiſteth alſo of two Parts, diſtinguiſhed by firſt and ſecond, as they lye in order; and is of neceſſary uſe for the finding of the Square-root, and of mean Proportions, as ſhall appear hereafter.

The fifth Scale is a Line of Tangents; This Scale abuts downwards upon the common Line *e, f,* and doth firſt contain the Artificial Tangents of the Quadrant from 0. *degr.* 35. *min.* to 45. *degr.* at the upper end of that Scale, and ſo if the Rule would permit, ſhould they be continued forward to 89. *degr.* 25. *min.* but becauſe the Diviſions of that Scale being inverted, will fall out to be the ſame with the former, they are to be noted and accounted backwards from 45. *degr.* at the upper end of that Scale to 89. *degr.* 25. *min.* at the lower end of the ſame; each degree thereof being ſubdivided into ſix Parts, and each of thoſe ſix Parts ſuppoſed to contain ten minutes.

The ſixth Scale is a Line of Sines: Upon this Scale ſhooting upwards upon the Common Line *e, f,* are deſcribed the Artificial Sines of the Quadrant from 0. *degr.* 35. *min.* to 90. *degr.* at the upper end of that Scale, each degree (upon our Rule) from 0. *degr.* 35. *min.* to 30. *degr.* being ſubdivided into ſix Parts, each Part repreſenting ten minutes, as thoſe of the Tangents; but from 30. *degr.* to 50. only into four Parts, each Part containing 15 minutes; from 50 to 70, into two Parts, each Part comprehending 30 minutes; from 70 to 85, into eaven degrees; and laſtly, from 85. *degr.* to 90, not divided at all, but ſuppoſed to be divided into five Parts, repreſenting thoſe five laſt degrees of the Quadrant.

The seventh Scale shooting downwards, is the whole Rule divided into 1000 *equal Parts;* It is hereafter called the *Scale of equal Parts*, and is of use for the Construction and Fabrick of the Great Line of *Numbers.*

The eighth Scale shooting upwards, is a Scale of 70 *degr.* 11 *min. of the Quadrant described according to* Mercator *and Mr.* Wright's *Projection:* It is hereafter called the *Scale of Latitudes,* and is to be used together with the *Scale of equal Parts;* and both of these taken together, are usually called the *Meridian Line,* and are of excellent use in Navigation, as shall be declared hereafter.

The ninth is the Scale of Inch-measure, viz. two foot thereof divided into 24 inches, and each inch into ten lesser Parts, counted both forwards and backwards, after the usual manner!

The tenth and last Scale consists of three several kinds, viz. *a Gage Line, a Line of Cords, and a Scale of Foot-measure:* The first of these being signed by the Letter *G,* is nothing else but seven inches divided into ten equal Parts, and those subdivided into ten lesser Parts, and is hereafter to be used for the ready discovery of the *equated* Diameter (and so by consequent of the Content) of any Wine, Beer, or Oyl Vessel: The next marked by the Letter *C,* is an ordinary Line of *Cords,* already sufficiently known, and of frequent use amongst Artists; the third and last, marked by the Letter F, is the *Scale of Foot-measure* being nothing else but a foot first divided into ten Parts, and those subdivided into ten lesser Parts, and so (by consequent) the whole foot supposed to be thereby divided into 1000 Parts.

At the end of these two Scales there is another double Scale placed, containing in length three inches *French*, whereof the uppermost shooting downwards, is a Scale divided into 60 Parts, and that shooting upwards into 100 Parts: The use of

these

these two Scales is for the ready reduction of Sexagenary minutes to Decimals, and of Decimal minutes to Sexagenaries, as shall appear hereafter.

CAP. II.

The Construction and Fabrick of the Lines described upon the Rule of Proportion.

1. TO *describe the* Line of Numbers, *having prepared a Rule of Silver, Brass, or Wood, (of what length you please) and caused it to be ruled according to the Pattern hereunto annexed, and also a Scale of* 1000 *equal Parts to be drawn, equal in length to your intended* Line of Numbers, *repair to the Table of Logarithms, and therein observing the first four Figures of the Logarithm of* 101, *beside the Index or Characteristick* (viz. 0043) *take with your Compasses the distance from the beginning of the said Scale of equal Parts to the said* 43 *Parts; This done, if you apply that extent of the Compasses upwards, from the beginning of the* Line of Numbers, *which you intend to make, the moveable Point of the Compasses, will fall upon the first* Centesme *of that Line: In like manner by the first four Figures of the Logarithm of* 102, *besides the Index* (viz. 0086) *you may mark the second* Centesine *of the same Line, and so consequently all the rest in their order.*

Example, If it were propounded to make a *Line of Numbers* equal to that of the first Scale, let there be a Scale of equal Parts made, equal in length to

that *Line*, such as the seventh Scale before described happens to be: then extending your Compasses from the beginning of that Scale of equal Parts to 0043, *viz.* to the Point *a*, apply that extent from the beginning of your Intended *Line of Numbers*; For, that done, the moveable Point of the Compasses will fall upon the first *Centesm* of that *Line*, *viz.* at the Point *e*: In like manner, the extent from the beginning of the Scale of equal Parts to 0086, *viz.* to the Point *c*, will mark out upon the intended *Line of Numbers* the Point *b*, representing the second *Centesm* of that *Line*, and so consequently the rest in order.

2. The *Line of Tangents* is framed much after the same manner; For, having before prepared a Scale of equal Parts sutable to that *Line*, (*viz.* consisting of half the length of the whole Line) *Repair unto the Table of Artificial Sines and Tangents, and therein finding the Artificial Tangent of* 0. *degr.* 40. *min. if (rejecting the Characteristick or first Figure thereof) you take off with your Compasses upon your foresaid sutable Scale of equal Parts (as before) the four first Figures of the same Tangent (viz.* 0658*) that extent being applied upwards from the beginning of the* Line of Tangents, *will cause the moveable Point of the Compasses to fall upon the Division, representing* 0. *degr.* 40.*min. In like manner the extent of* 1627 *(the second, third, fourth, and fifth Figures of the Tangent of* 0. *degr.* 50. *min.) will guide to mark out the same* 0. *degr.* 50.*min. upon the same Line:* And so proceeding you may readily describe all the rest, as they follow in order.

3. The *Line of Sines* may be drawn in all Points, as the *Line of Tangents*, if you use the second, third, fourth and fifth Figures of the *Artificial Sines*, as you are before directed to use those of the *Tangents*. And here *note*, that the Line before called the *Mean Line of Numbers*, and these Lines of *Tangents* and *Sines* are all of them framed by one and the same Scale,

Scale, and are alſo hereafter to be uſed together in the reſolution of *Plain Triangles*, the Scale of equal Parts or *Radius*, by which they are made, being in each of them twice repeated.

4. The *Meridian Line* being framed by the ordinary Table of Meridional Degrees, and the making of the *Line of Cords* being obvious to every mean Practitioner in the Mathematicks, I ſhall not need to trouble you with their Conſtruction. The other Scales alſo, which conſiſt of equal Parts, will not need any farther deſcription.

CAP. III.

Numeration upon the Rule of Proportion.

PROBL. I.

A whole Number being given, to find the Point where the ſame is repreſented upon the Line of Numbers.

Find amongſt the Figures, by which the Primes are diſtinguiſhed, the firſt Figure of the Number given, and for the ſecond Figure thereof count from the beginning of the Prime, unto which the firſt Figure directs you, ſo many Tenths as that Figure hath Unites; Then for the third Figure count from the laſt Tenth ſo many Centeſms as that

third Figure hath Unites: And so likewise for the fourth Figure count from the last Centesme so many Millains as the same fourth Figure hath Unites: This done, you shall at last fall upon the Point where the Number propounded is represented upon the Line of Numbers.

Example, The Number given being 1728, the first Figure thereof (*viz.* 1.) leads me unto the first *Prime*, designed by the Figure 1, within which *Prime* counting seven *Tenths* for the second Figure, and from the seventh *Tenth* two *Centesmes*, for the third Figure, and from the second *Centesme* eight *Millains* for the fourth Figure; at last I find the Number given to be represented upon the first Part of the Great Line of Numbers at the Point *h*: So likewise is the Number 27 found at the Point *k*, the Number 542 at the Point *l*, and 3345 at the Point *m*, &c.

From hence follow these Corollaries:

1. *The Figure which any Number given hath towards the right hand, besides the first four Figures towards the left hand, are not expressed upon the Rule:* And therefore if the Number given were 172 845, it would be likewise represented at the Point *h*: Howbeit, that uncertainty causeth no inconvenience in the use of the Rule, as shall more plainly appear hereafter.

2. *The Figures by which the Primes are distinguished (in reference to one and the same Number) retain always one and the same value.*

Example, In searching the Number 1728, conceiving the Figure prefixed at the beginning of the first *Prime* (*viz.* 1.) to have the value of *Thousands*, the Figure prefixed before the second *Prime* (*viz.* 2.) ought also to be esteemed to have the value of *Thousands*, and so of the rest in their order: for, according to the same reason that *h* represents 1728, the Point *n* will represent 2000, the Point *p* 3000, &c.

3. *The Numbers, which have only the simple value of Unites as* 1. 2. 3, 4. *&c. and these which after the first*

Figure

Figure have nothing but Cyphers, as 10.100.1000.20.200. 2000. *&c. are all represented at the same Points.*

So 1.10.100.1000.*&c.*may be all represented at the beginning or end of the *Line* 2. 20. 200. 2000. *&c.* at the beginning of the second *Prime:* 3. 30. 300. 3000. *&c.* at the beginning of the third *Prime, &c.*

4. *The Numbers, which being composed of three Figures have a Cypher in the middle, are found betwixt the beginning of the Prime, unto which they belong, and the first Tenth of the same Prime.*

So 405 beginning by the Figure 4, (and therefore to be sought for in the fourth *Prime)* is represented at the Point *o.*

5. *The Numbers, which being composed of four Figures, have now Cyphers in the middle, are represented betwixt the beginning of the Prime, unto which they belong, and the first Centesme of the same Prime:* So 1005 is found at the Point *q.*

6. *When the Line of Numbers is repeated, and for that cause consisteth of several Parts; the first Part thereof is in value a degree less than the second, and the second a degree less than the third,* &c.

So upon the Mean Line of Numbers, if you conceive 10 at the upper end thereof to represent 100, the Figure 1 in the middle (or which is all one, at the beginning of the second Part) will represent 10, and 1 at the lower end of that Line (or which is all one, at the beginning of the first Part) will represent 1: But if 10 at the upper end thereof shall be conceived to bear but the value of 10, the Figure 1 in the middle shall have the value of one, and one at the lower end the value of $\frac{1}{10}$, and 2 the value of $\frac{2}{10}$, *&c.* In like manner, if 10 at the upper end represent 1, the Figure 1 in the middle must represent $\frac{1}{10}$, and 1 at the lower end $\frac{01}{100}$, *&c.*

PROBL. 2.

To find a Fraction or broken Number upon the Line of Numbers.

THe Fractions, which are to be found upon the Line of Numbers, ought always to be Decimals, *viz.* ought always to have for their Denominators the Figure 1, with nothing but Cyphers towards the right hand, such as are $\frac{125}{1000}$ $\frac{25}{100}$ $\frac{5}{10}$ $\frac{75}{100}$ or the like, which may otherwise be written thus, .125.25,5,75, and are equivalent to $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ and $\frac{3}{4}$: And therefore if the Fractions propounded be not Decimals, they ought to be reduced to such: For, that done, they may be discovered in all Points as whole Numbers are found out upon the Line, which may be plainly understood by the *Examples* produced in the sixt *Corollary* of the last Problem.

PROBL. III.

To find a Mixt Number upon the Line of Numbers.

F*Irst find by the first Problem aforegoing the Point representing the whole Parts of the Number given, and then afterwards the Fraction or broken Parts thereof in the Ranks that follow.*

Example, a Line that hath the length of 17 foot and $\frac{28}{100}$ of a foot (which may more conveniently be

be written thus, 17, 28) being propounded; first, I find the whole Parts thereof, *viz.* 17) represented at the Point *r*, and after counting two Centesms, and then eight Millains, at last I find the Number given to be represented at the Point *h*. In like manner if the Number propounded were 172. 8, or 1. 728, it would be still represented at the same Point.

PROBL. IV.

Any Point of the Line of Numbers being assigned, to find the Figures represented at the same Point.

Take the Figure prefixed at the beginning of the Prime, within which the Point is propounded, for the first of the Figures required; then shall the second Figure required be composed of so many Unites as there are Tenths intercepted betwixt the beginning of the same Prime and the Point given. In like manner shall the third Figure required have so many Unites as there are Centesms comprehended betwixt the last of those Tenths and the said Point: And so likewise shall the fourth Figure consist of so many Unites as there are Millains between the last Centesm and the Point given.

Example, If the Point *h* were propounded, because that Point is situate within the Prime, before which the Figure 1 is prefixed, I take the Figure 1 for the first of those required; and then finding seven Tenths betwixt the beginning of that Prime and the Point given, I set down 7 for the second: And so proceeding and finding two Centesms betwixt the last Tenth and the said Point, I take 2 for the third Figure: And lastly, conceiving eight Millains to be comprehended between the last Cen-

tesm

tesine and the Point given, I take 8 for the fourth Figure required: This done, I conclude, that the Figures represented at the Point propounded, are 1728. In like manner the Point *q* being given, I take 1 for the first Figure; but here because I find no Tenths betwixt the beginning of that Prime and the Point given, I write a Cypher in the second place; and there also finding no Centesines, I write also a Cypher in the third place; And then at last finding the Point propounded in the middle of a Centesine (which is supposed to be divided into ten Millains) I annex in the fourth place 5: This done, the Figures represented at the Point given will be found 1005.

PROBL. 5.

An Ark or Angle being propounded to find upon the Rule of Proportion the Point which represents the Tangent of the same Ark or Angle.

If the Ark or the measure of the Angle exceeds not 45 degrees, search the degrees of that Arke or Angle upon the Line of Tangents, mounting upwards from the lower end of that Line towards the upper end of the same.

So the Tangent of an Ark or Angle, which consists of 15 degrees, is represented at the Point *a*: of 25 degrees at the Point *h*, &c.

But if the Ark or measure of the Angle exceeds 45 degrees, look the degrees thereof, descending downwards from the upper end of the Line towards the lower end of the same: So the Tangent of 65 degrees is found at the

the Point *b*, of 75 degrees at the Point *a*, *&c.*

And if the Ark or Angle propounded (besides the whole degrees) is also composed of certain minutes, find first the whole degrees, and after that, betwixt the last degree found, and the next that follows, take so many of the Parts which may amount to the minutes given accounting each of the Parts contained betwixt the two degrees for ten minutes: So the Tangent of 22 *degr.* 45 *min.* is found at the Point *d*, and the Tangent of 72 *degr.* 45 *min.* at the Point *a*. And therefore *e converso*, if the Points *d* and *t* were given upon this Line, the degrees and minutes represented by them, would be 22 *degr.* 45 *min.* and 72 *degr.* 45 *min.* &c.

PROBL. 6.

An Ark or Angle being propounded, to find upon the Rule of Proportion the Point, which represents the Sine of the same Ark or Angle.

F*Ind upon the Line of Sines the degrees of the Ark or Angle given, and you have your desire:* So the Sine of the Ark or Angle of 22 *degr.* is represented at the Point *r*.

But if the Ark or Angle given have also minutes annexed, first search the whole degrees given, and then between that degree found and the next that follows, take so many Parts as you have minutes propounded, conceiving the distance betwixt each degree, and the next that follows to comprehend 60 minutes.

So the Sine of 22 *degr.* 45 *min.* is found at the Point *u*; of 42 *degr.* 50 *min.* at the Point *q*; of 52 *degr.* 45 *min.* at the Point *k*, *&c.* And therefore here also

alſo *e converſo*, if the Points *u*, *q*, and *e* were aſſigned upon this Line, the degrees and minutes repreſented by them would be 22 *degr.* 45 *min.* 42 *degr.* 50 *min.* and 52 *degr.* 45 *min.* &c.

CAP. IV.

The uſe of the Rule of Proportion in Arithmetick.

IN *Arithmetick* there are three ſeveral ſorts of Proportion, *Arithmetical*, *Geometrical*, and *Muſical*. *Arithmetical*, when divers Numbers being compared together retain amongſt themſelves equal differences, as theſe, 2. 4. 6. 8. *&c.* And this is either *continued*, as in the Numbers before produced, or in theſe, 3. 6. 9. 12. 15, *&c.* which is alſo called *Arithmetical* Progreſſion, or a Rank of Numbers *Arithmetically* proportional; or diſcontinued, as in theſe, 2. 4. 10. 12, or the like. *Geometrical* Proportion is, when divers Numbers being compared together differ amongſt themſelves according to the ſame rate or reaſon, as theſe, 2. 4. 8. 16. *&c.* For here, as 2 is half 4, ſo is 4 half 8, and 8 half 16: this is likewiſe either *continued*, as in thoſe before propounded, or in theſe, 1. 3. 9. 27. 81. *&c.* or the like, which is alſo called *Geometrical* Progreſſion, or a Rank of Numbers *Geometrically* proportional: Or *diſcontinued*, as in theſe, 2. 4. 16. 32; for as 4 is double 2, ſo is 32 double 16, but ſo is not 16 being compared with 4. *Muſical* Proportion is that which doth as it were proceed from both the former, as when three Numbers or Terms being propounded, the firſt bears the ſame Proportion to the third, that the difference betwixt

betwixt the first and the second bears to the difference betwixt the second and third, as in these, 3. 4. 6, for here, as 3 is half 6, so is 1, the difference betwixt 3 and 4, to 2, the difference betwixt 4 and 6. So likewise 2. 3. 6. and 10. 16. 40. are said to be Numbers *Musically* proportional: For, in the first of these two last Examples, as 2 is to 6, so is 1 to 3; And in the others, as 10 is to 40, so is 6 to 24. Thus have I here thought fit briefly to remember the *Reader* of the several kinds of Proportion, which he doth usually find in the Writings of those that treat of *Arithmetick*; to the end that the *Problems* which follow both in *Arithmetick* and *Geometry* may be the better understood.

PROBL. I.

Two Numbers being given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

Extend the Compasses upon the Line of Numbers from one of the Numbers given to the other; this done, if you apply the same extent (upwards or downwards) from either of the Numbers propounded, the moveable Point of the Compasses will fall upon the third proportional required: And so the same extent being applyed the same way from the third, the moveable Point of the Compasses will fall upon the fourth proportional, and from the fourth upon the fifth, &c.

Example, Let it be propounded to find a third proportional to these two Numbers 2 and 4, which may bear the same Proportion to 4, that 4 bears to

to 2; First, I Extend the Compasses upon the first Part of the Mean Line of Numbers from 2 to 4; this done, if I apply that extent outright from 4 upwards, the moveable Point of the Compasses will fall upon 8 the third Proportional required; and being applied the same way from 8, the movable Point will rest upon 16, the fourth Proportional; and from 16 to 32, the fifth; and from 32 to 64, the sixth Proportional. But now if you would yet continue the Progression farther, and so find the next Proportional to 64 (because the movable Point in that case will fall beyond the Line) apply that extent the same way from 64 in the first Part of that Line; which done, the movable Point of the Compasses will then fall upon 128, the seventh Proportional; and so proceeding farther you may find 256, the eighth 3512, the ninth, *&c.*

Contrariwise, if it were required to find a third Proportional to the same Number 2 and 4, which may bear the same proportion to 2, that 2 bears to 4; extend the Compasses upon the second Part of the Mean Line of Numbers from 4 to 2 downwards; this done, if you apply that extent from 2 the same way (*viz.* downwards) the movable Point will fall upon 1, the third Proportiona required; And from 1 upon $\frac{5}{10}$ or .5, by the last Corallary of the third Chapter, and from .5 to .25, by the same Corallary, *&c.*

In like manner, if the two Numbers given were 10 and 9, the Compasses being extended downwards from 10 at the upper end of the same Line of Numbers to 9, and that extent applied from 9 the same way, the movable Point of the Compasses will rest upon 8.1, the third Proportional (for the given Numbers being 10 and 9, common sense tells me that it cannot be 81, and therefore ought to be 8.1) and from 8.1 the movable Point will fall upon

on 7.29, the fourth Proportional, *&c.* So likewise if the Numbers propounded were 1 and 9, conceiving 10 at the upper end of the Line to represent 1, extend the Compasses from thence to 9, which extent being applied downwards from 9, will cause the movable Point of the Compasses to fall upon 81, the third Proportional, and from 81 upon 729, the fourth Proportional, *&c.* And therefore *note* hence, that 1 at the beginning, 1 in the middle, and 10 at the end of the Line, are all arbitrary Points, and may each of them represent sometimes 1, sometimes 10, sometimes 100, sometimes 1000, *&c.* as the terms by which you are to work, shall require, according to the third Corollary of the third Chapter.

Neverthelefs neither do the *Examples* before produced, nor those, which shall follow in the ensuing Problems at all cross that which hath been formerly taught in the second Corollary of the third Chapter: For, in the last *Example*, the end of the Line in regard of the first term given (*viz.* 1) hath the single value of an Unite, but in respect of the second term 9 it challengeth the value of 10; and in reference to the third Number 81, the value of 100, *&c.*

Lastly, if the Numbers given were 10 and 12, the third Proportional upwards would be 14.4, the fourth 17.28, *&c.* and the Number 1 and 12 being propounded, the third Proportional upwards (as before) will be 1443, the fourth 1728, *&c.*

The like Operations may be also performed (and that much more exactly) upon the great Line of Numbers: For *Example*, 1 and 4 being given, I desire to know a third, a fourth, a fifth, *&c.* Geometrically Proportional: To perform this, extend the Compasses upon that Line across from 1 at the beginning of the second Part thereof unto 4 upon the first part of the same; which done, that extent being

ing applied the ſame way, (*viz.* upwards and acroſs) will reach from 4 upon the firſt Part, unto 16 upon the ſecond, and from thence to 64 upon the firſt Part again, *&c.*

PROBL. 2.

One Number being given to be multiplied by another Number given, to find the Product.

E*Xtend the Compaſſes upon the Line of Numbers from 1 unto the Multiplicator; This done, if you apply that extent the ſame way from the Multiplicand, the moveable Point of the Compaſſes will fall upon the Product required.*

1. *Example*, Let the Multiplicator given be 25, and the Multiplicand 30: Here if you extend the Compaſſes upon any of the Lines of Number from 1 unto 25, and then apply that extent the ſame way from 30, the moveable Point of the Compaſſes will fall upon 750, the Product required. So 1. 728, and 25. 6 being propounded to be multiplied, the Product will be found 44. 2.

2. *Example*, The two Numbers given being 45 and 25, I extend the Compaſſes upon the ſecond Part of the Mean Line of Numbers from 1 to 25; Then (becauſe, if I apply that extent the ſame way from 45 upon the ſame Part of that Line, the moveable Point will fall beyond the Line) I apply the ſame extent the ſame way from 45 in the firſt Part thereof; which done, the movable Point will fall upon 1125, the Product deſired: So the two Numbers given, being 1.728, and 64.5, the Product required will be 111.4.

3. *Example*,

3. *Example*, If 75 and 35 were given to be multiplied, the Compaſſes ought to be extended downwards from 1 to 75, in the firſt Part of the Mean Line of Numbers, or (which is all one) from 10 at the upper end of that Line to 75; for, that extent being applied the ſame way from 35, will cauſe the movable Point of the Compaſſes to fall upon 2625, the Product required.

4. *Example*, If it were required to find the Content of a piece of Ground 8.75 Perches long, and 6.45 broad; becauſe this queſtion is reſolved by multiplying the length by the breadth, I extend the Compaſſes from 10. at the top of the Line to 8.75; then applying that extent the ſame way from 6.45, the movable Point will fall upon 56.4, the Content required, *viz.* 56 Perches and $\frac{4}{1}$. or .4 of a Perch.

And here you may obſerve, that theſe laſt *Examples*, and thoſe that are like unto them, may likewiſe be performed in working upwards; But in ſuch caſes to ſhun too great an extent of the Compaſſes, it is better to begin the Operation from 10 at the top of the Line, and ſo to deſcend downwards according to the Inſtructions before delivered: For, (take this for a General Rule, once for all, that) *All Operations, which are wrought upon the Rule of Proportion, are beſt performed, when the legs of the Compaſſes have the leaſt extenſion.*

Again, becauſe this Problem of *Multiplication*, as alſo (for the moſt part) all the reſt that follow, are reſolved by the finding out of a fourth Number *Geometrically* proportional to three other Numbers given, we will therefore here inſert this other Advertiſement: Whenſoever queſtion is made of finding a fourth proportional to three ſuch Numbers given, for the better conveniency of working upon the Rule, the order of the ſecond and third terms may be changed, ſo that always care be taken, that the

firſt

first Number may still retain the first place: For *Example*, you may say, as 1 is to 25, so is 30 to 750; or as 1 is to 30, so is 25 to 750. And this Rule is diligently to be observed in Multiplication, Division, the Rule of three direct, the resolution of the Plane and Spherical Triangles, and generally in all Questions of such like Proportions; to the end that in working upon the *Rule of Proportion* we may always avoid too great an exension of the Compasses, and by that means perform the Work the more exactly.

Lastly, here observe, that Multiplication, and all other Questions hereafter produced, which may be wrought upon the Mean Line of Numbers, may likewise be performed upon the Great Line of Numbers (and that much more exactly) by working either outright or across, as the Questions propounded shall require; which (I well hope) I may hereafter leave to the discretion of the ingenious Reader to discover, without any further instruction, they being (indeed) but one and the same *Instrument* represented in differing postures.

PROBL. 3.

A Number being propounded to be divided by another Number, to find the Quotient.

Extend the Compasses upon the line of Numbers from the Divisor to 1; This done, if you apply that extent the same way from the Dividend, the movable Point will fall upon the Number of the Quotient.

1. *Example*, Let 750 be the Number given to be divided

divided by 25, the Diviſor: I extend the Compaſſes downwards from 25 to 1; then applying that extent the ſame way from 750, at laſt the movable Point will fall upon 30, the Quotient required.

2. The Number 1125 being given to be divided by 25; I extend the Compaſſes downwards from 25 to 1, then applying that extent the ſame way from 1125, the movable Point will fall upon 45, the Quotient required. The ſame Quotient will alſo be found, if changing the terms you firſt extend the Compaſſes from 25 to 1125, and then apply that extent from 1; for ſo alſo ſhall the movable Point fall upon 45, as before; according to the obſervation made in the laſt Problem: In like manner 111.4 being propounded, to be divided by 1.728, the Quotient will be found 64.5.

3. The Number 2625 being propounded to be divided by 75; extend the Compaſſes upwards from 75; in the firſt Part of the Mean Line of Numbers to 1, or (which is all one) from 75 in the ſecond Part thereof to 10 at the top of the Line; This done, if you apply that extent the ſame way from 2625, the movable Point will from thence reach to 35, the Quotient required: So likewiſe 56.4 being given to be divided by 8.75, the Quotient will be 6.45.

Now to diſcover of how many Figures any Quotient ought to conſiſt, it will be neceſſary to obſerve how many times the Diviſor may be written under the Dividend according to the Rules of Diviſion; for, of ſo many Figures ſhall the Quotient be compoſed: for *Example*, 12231 being given to be divided by 27; becauſe the Diviſor 27 may (according to the Rules of Diviſion) be written three times under the Dividend 12231 (as may appear by this *Example*) I ſay, that the Quotient, which is produced by the Diviſion of 12231 by 27 conſiſts of three Figures

12231
27 . .

For

For, having extended the Compaſſes downwards in the ſecond Part of the Mean Line of Numbers from 27 (the Diviſor) to 12231 (the Dividend) and applied that extent the ſame way from 1, the moveable Point will fall in the firſt Part upon 453, the Quotient of 12231 divided by 27.

PROBL. 4.

To three Numbers given to find a fourth in a direct Proportion.

Extend the Compaſſes from the firſt Number or Term given, unto the ſecond; which done, that extent being applied the ſame way from the third Term, will cauſe the movable Point to fall upon the fourth Term required.

Example, If the circumference of a Circle, whoſe Diameter is 7, be 22; what circumference will a Circle have, whoſe Diameter is 14? Extend the Compaſſes upwards upon the Mean Line of Numbers from ſeven in the firſt Part thereof, unto 14 in the ſecond; This done, that extent being applied the ſame way from 22, will make the movable Point reſt upon 44, the circumference required.

Or otherwiſe downwards; The circumference of a Circle being 22, and the Diameter thereof 7, how much ſhall the Diameter of a Circle be, whoſe circumference is 44? Extend the Compaſſes downwards from 22 in the ſecond Part, to 7 in the firſt; which done, that extent being applied the ſame way from 44, will reach to 14, the Diameter ſought for.

PROBL.

PROBL. 5.

To three Numbers given, to find a fourth in an inversed Proportion.

Extend the Compasses upon the Line of Numbers from the first of the Numbers given to the second, having both the same Denomination; this done, if that extent be applied quite backwards from the third given Number, the movable Point will fall upon the fourth Number you look for.

Example, If 60 Pioners can make a Trench of a certain length and breadth in 45 hours, how long will it be before 40 men can make such another? Extend the Compasses from 60 to 40 (those Terms having both the same Denomination, *viz.* of men.) This done, that extent being applied backwards from 45, will reach to 67. 5, the fourth Number you look for; I conclude therefore that 40 men will perform as much in 67 hours and an half, as 60 men will do in 45 hours.

PROBL. 6.

To three Numbers given, to find a fourth in a double Proportion.

THe use of this Problem appears chiefly in Proportions of Lines to Superficies, or of Superficies to Lines.

Now

Now if the Denomination of the firſt and ſecond terms be of Lines, *Extend the Compaſſes upon the Line of Numbers, from the firſt term to the ſecond ; this done, that extent being applied twice the ſame way from the third term will cauſe the movable Point to fall upon the fourth term required.*

Example, If the Content of a Circle whoſe Diameter is 14 inches, be 154, what will the Content of a Circle be, whoſe Diameter is 28 ? Here 14 and 28 having the ſame Denomination (*viz.* of Lines) I extend the Compaſſes from 14 to 28 ; then applying that extent the ſame way from 154, the movable Point will firſt fall upon 308, and from hence upon 616, the Content deſired.

But if the firſt two terms have the Denomination of *areas* or Contents, and the *quæſitum* be a Line, this is the Rule: *Extend the Compaſſes upon the Mean Line of Numbers from the firſt term to the ſecond ; this done, that extent being applied the ſame way upon the Great Line of Numbers from the third term, will cauſe the movable Point to fall upon the fourth term required.*

Example, If the Diameter of a Circle, whoſe *area* is 154, be 14 ; what Diameter will a Circle have, whoſe *area* is 616 ? Extend the Compaſſes upon thee Mean Line of Numbers from 154 to 616 ; which done, that extent being applied the ſame way upon the Great Line of Numbers from 14, will reach to 28, the Diameter required.

PROBL. 7.

To three Numbers given, to find a fourth in a tripled Proportion.

THe uſe of this Problem appears in the proportion of Lines to Solids, & *contra.*

If

If therefore the first and second Terms have the denomination of Lines, *Extend the Compasses upon the Line of Numbers from the first Term to the second; this done, and that extent applied three times the same way from the third Term; will cause the movable Point at last to fall upon the fourth Term required.*

If an Iron Bullet, whose Diameter is 4 inches, weighing 9 pounds, what is the weight of another Iron Bullet, whose Diameter is 8 inches? Extend the Compasses from 4 to 8! which done, and that extent applied the same way three times from 9, the movable Point will first fall upon 18, then from 18 upon 36, and at last from 36 upon 72, the weight required.

But if the first two Terms be weights or contents of Solids, and a Line is sought for: *Extend the Compasses upon the Little Line of Numbers from the first Term to the second; This done; and that extent applied the same way upon the Great Line of Numbers from the third term will cause the movable Point of the Compasses to fall upon the fourth Term required.*

If the side of a Cube weighing 72 pounds be 8 inches, how many inches is the side of a Cube that weighs 9 pound? Extend the Compasses downwards upon the Little Line of Numbers from 72 to 9; that done, and the same extent applied the same way upon the Great Line of Numbers from 8, will cause the movable Point to fall upon 4, the side required.

PROBL. 8.

Betwixt two Numbers given to find a Mean Arithmetically Proportional.

THis Problem may be performed without the help of the *Rule of Proportion*: Nevertheless,

 because

because it conduceth to the resolution of the next insuing Problem, I insert it in this place, and give this Rule for it:

Add half the difference of the given Terms to the lesser of them: for, that aggregate is the Arithmetical Mean required.

Example, Let 10 and 40 be the Terms given: here, if you substract the one out of the other, their difference will be found 30. whose half (15) being âdded to 10, the lesser Term, their sum (25) is the Arithmetical Mean you look for.

PROBL. 9.

Betwixt two Numbers given, to find a Mean Musically proportional.

Boetius (*Lib. 2. Arith. cap.* 38.) hath this Rule for it: *Differentiam terminorum in minorem terminum multiplica, & post junge terminos, & juxta eum, qui inde confectus est, committe illum numerum, qui ex differentiis & termino minore productus est, cujus cum latitudinem inveneris, addas eam minori termino, & quod inde colligitur medium terminum pones.* Multiply the difference of the Terms by the lesser Term, and add likewise the same Terms tegether: this done, if you divide that Product by the Sum of the Terms, and to the Quotient thereof add the lesser Term that last Sum is the Musical Mean desired.

Or shorter thus:

Divide the Product of the given Term by their Sum: for, this done, the Quotient doubled is the mean required. So the Numbers given being 6 and 12, I say 12 multiplyed

tiplyed by 6 make 72, which divided by 18 the Sum of 12 and 6) leaves 4 in the Quotient, whoſe double (8) is the Muſical Mean you look for. This Problem therefore may be performed by the ſecond and third aforegoing! or yet otherwiſe thus:

Find the Arithmetical Mean betwixt the Number given and then the Analogy will be this.

As the Arithmetical Mean found is to the greater Extreme: ſo is the leſſer Extreme to the Muſical Mean required.

Example, 10 and 40 being propounded, the Arithmetical Mean betwixt them (by the laſt Problem) is 25: I ſay then, As 25 is to 40, ſo is 10 to 16, the Muſical Mean deſired: the Term therefore here ſought for may be diſcovered by the fourth Problem aforegoing.

And here (I conceive) it will not be amiſs to obſerve, that by this laſt Rule, having any two Numbers propounded, you may interject two other Numbers betwixt them! in ſuch ſort that they four being in ſeveral relations compared one with another, may contain in them all the three Proportions abovementioned, which kind of Harmony *Boetius (lib. 2. cap. ult.)* calls *Maxima & perfecta ſymphonia:* So in the Numbers before mentioned 10, 16, 25, and 40; if you compare 10, 25, and 40 together, there ſhall you find *Arithmetical* Proportion; if 10, 16, and 40 together, there Harmony, or *Muſical* Proportion; if all of them together, there have you *Geometrical* Proportion diſcontinued: For as 10 to 16, ſo 25 to 40. And this is that *Harmony* which the ſame *Boetius* (in the ſame place) affirmeth to have *Magnam vim in Muſici modulaminis temperamentis, & in ſpeculatione naturalium quæſtionum:* Great force in the compoſure of Muſick, and in the diſcovery of the ſecrets of Nature: And therefore he alſo averreth in another place (*viz. lib.* 1. *cap.* 2.) that *the reaſon of Numbers was the chiefeſt Rule accor-*

ding to which Almighty God framed the World: According to that testified of the Wisdom of God (in the Wisdom of *Sol.* cap. 11. v. 20.) *Thou hast ordered all things in Measure; and Number; and Weight.* The Statists also and Politicians fetch much from these three Proportions for the regular direction of a well governed Commonwealth, as may be easily collected out of their Writings, and is learnedly proved by *Bodin* in the last Chapter of his Commonwealth.

PROBL. 10.

Betwixt two Numbers given, to find a Mean Geometrically Proportional.

Extend the Compasses upon the Mean Line of Numbers from one of the Numbers given to the other; this done; and the same extent applied upon the Great Line of Numbers from either of those Numbers towards the other; the movable Point will fall in the middle betwixt them; viz. *upon the Point representing the Mean Proportional required.*

Example; 8 and 32 being propounded, the Mean Proportional between them will be found 16: For if I extend the Compasses upon the Mean Line of Numbers, from 8 in the first part thereof to 32 in the second, and afterwards apply that extent upon the great Line of Numbers from 8 towards 32, the movable Point will fall upon 16, the Mean Proportional demanded; for as 8 is to 16. so is 16 to 32: so the Mean betwixt 6.4, and 14. 4, is 9. 6, *&c.*

PROBL. 11.

PROBL. 11.

Betwixt two Numbers given to find two Means Geometrically Proportional.

Extend the Compasses upon the Little Line of Numbers from one of the Numbers given to the other: this done, and that extent applied upon the Great Line of Numbers from either of those Numbers towards the other; will cause the movable Point to fall first on the third part of the distance between them; viz. *upon the Point representing one of the Mean Numbers required; and being applied again the same way; will at last rest upon the other Proportional you look for.*

Example; Let 8 and 27 be the two Numbers between which two Mean Proportionals are desired. First, I extend the Compasses upon the Little Line of Numbers upwards from 8 to 27: then applying that extent twice upon the Great Line of Numbers from 8 towards 27, I find the movable Point to fall first upon 12, and then upon 18, which are the two Means you desire to know: for as 8 is to 12, so is 12 to 18, and 18 to 27.

PROBL. 12.

To find the Square-Root of any Number under 1000000.

The *Extraction* of Roots, which is accounted the hardest Lesson in *Arithmetick*, is performed by

he help of this *Instrument* with greatest ease and dexterity: for, whereas the *Problems* before premised, as also those that follow, cannot well be expedited without the joynt use of the *Rule* and *Compasses* together, these of the *Extraction* of the *Square* and *Cube* Roots may be resolved only by *Inspection* without any trouble at all, or ayd of Compasses: so that a man either riding or going in haste may immediately read upon the *Rule* the Root of any *Square* or *Cube* Number propounded: which compendious way of *Extraction* cannot choose but prove to be of admirable use, especially in questions that concern Military Orders, as shall more plainly appear hereafter. Wherefore to extract the *Square-Root* proceed thus:

1. *When the Figures of the Number given are even,* viz. *when the Number consists of two, four, or six Figures, look the same Number in the first part of the Mean Line of Numbers: which done, just at the same Point shall you likewise find upon the Great Line of Numbers the Square-Root you look for.*

Example, 264196 being propounded, the Square-Root thereof will be found 514: for I find the Number 264196 represented in the first part of the Mean Line of Numbers at the Point *x*, and at the same Point upon the second part of the Great Line of Numbers I observe 514, the Square-Root required.

2. *When the Figures of the Number given are odd,* viz. *one, three, or five, search the same Number in the second part of the Mean Line of Numbers: which done, just at the same Point upon the Great Line of Numbers shall you find also the Square-Root demanded.*

Example, 144 being propounded, I demand the Square-Root thereof: that Number I find to be represented in the second part of the Mean Line of Numbers at the Point *s*, and just there also upon the Great Line of Numbers I discover 12, which is the

the Square-Root of the Number propounded. So likewise is 144 the Square-Root of 20736.

PROBL. 13.

To extract the Cube-Root of any Number under 1000000000.

1. *When the Number propounded consists of one, four, or seven Figures, find it in the first part of the Little Line of Numbers: that done, at the same Point upon the first part of the Great Line of Numbers, you shall find the Cube-Root you look for.*

Example. Let the Number given be 1728 whereof the Cube-Root is required: I find that Number in the first part of the Little Line of Numbers at the Point *t*, and at the same Point upon the Great Line of Numbers I also discover 12, the Cube-Root desired: In like manner is 12,52 the Cube-Root of 1950, and 144 the Cube-Root of 2985984.

2. *When the Number given consists of two, five, or eight Figures, search it in the second part of the Little Line of Numbers, and that proceeding as before, you shall have your desire.*

Example, If 14348907 were given, the Root thereof would be found 243: for, that Number being found in the second part of the Little Line of Numbers at the Point *u*, just at the same Point upon the Great Line I also find 243, the Cube-Root required.

5. *When the Number propounded consists of three, six, or nine Figures, look for it in the third part of the Little Line of Numbers: for so likewise at the same Point upon the Great Line will appear the Root required.*

So the Number 159220088 being found in th first part of the Little Line of Numbers at the Poin z, his Cube-Root is there likewise found upon th Great Line of Numbers to be 542: And the Cub- Root of 159220. is found to be 54. 2, *&c.*

The order of finding out the Cube-Numbers upon the several parts of the Line may be fitly expressed by this Figure

1	2	3
1	2	3
4	5	6
7	8	9

CAP. V.

The Use of the Rule of Proportion, in Geometry, viz.

In the Dimension,

1. Of Plain Triangles.

PROBL. 1.

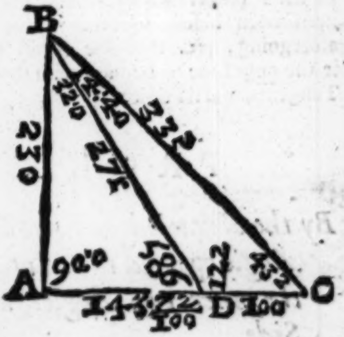
The three Angles and one Side being known, to find the other two Sides.

TO resolve this Problem this is the *Analogy.* As the Sine of the Angle opposed to the side known

known is to the parts of the same side: so is the Angle opposed to one of the sides unknown, to the parts which measure that side: And therefore

Extend the Compasses across from the Sine of the Angle opposed to the side known; to the same side; found upon the Mean Line of Numbers: then applying that extent the same way from the Sine of the Angle opposed to one of the sides required; the movable Point will fall upon the parts which measure that required side.

Example In the Triangle *C, B; D;* let the Angle *C* be 43 *degr.* 20 *min.* the l Angle *D* 122 *d.* and by consequent the Angle *B* (being the Complement of the two other Angles to 180 *d.* or two right Angles) 14 *degr.* 40 *min.* and let the side *D; C,* being 100 paces represent the distance between the two stations *D* and *C*: I demand then the distance between *C* and *B*: Extend the Compasses across from 14 *degr.* 40 *m.* upon the Line of Sines to the middle of the Mean Line of Numbers representing 100, then that extent being applied the same way from 122 *d.* upon the Line of Sines or (which is all one) from 58 *degr.* (for by the Rules of *Trigonometry* the Sine of an obtuse Angle and that of his Complement to 180 is one and the same Line) will cause the movable Point to fall upon 135, and so many paces is the distance required: In like manner, the extent being applied the same way from 43 *d.* 20 *m.* upon the Line of Sines, the movable



Point will fall upon 271, the parts of the fide *D*, *B*.

Or otherwife, by changing the Terms of the *Analogie*, thus:

Extend the Compaffes outright upon the Line of Sines from 14 *d*. 40 *m*. to 58 *d*. then applying that extent the fame way upon the Line of Numbers from 100, the moveable Point will reft upon 335, the diftance required: fo likewife the Compaffes being extended outright upon the Line of Sines from 14 *d*.40 *m*.to 43 *d*. 20 *m*. and that extent applied the fame way upon the Line of Numbers from 100, the moveable Point willfall upon 271, the parts of the fide *D*, *B*.

And here obferve, that not only this prefent Problem, but alfo all thofe that follow (which concern the refolution of Triangles) may be refolved two manner of wayes, *viz*. by working either outright or acrofs, except fome few, which we intend to mark in their proper places. Remember likewife what hath been before touched in the fecond Chapter aforegoing, *viz*. that the Mean Line of Numbers ir the only Line to be ufed with thefe of Sines and Tangents, and no other.

PROBL. 2.

By the Knowledge of two Sides and an Angle oppofed to one of them, to find the other two Angles and the third Side.

THis is the *Inverfe* of the laft Problem: for, as the fide oppofed to the given Angle is, to the Sine of the fame Angle: fo is the other fide known,

known, to the Sine of the Angle thereunto opposed: And therefore

Extend the Compasses across from the parts of the side opposed to the Angle known, unto the Sine of the same Angle: then that extent being applyed the same way from the parts of the other known side, will cause the moveable Point to fall upon the Sine of the Angle required.

So in the foresaid Triangle *C, B, D*, the side *C, B*, being 335, the Angle *D* (opposed thereunto) 122 *d.* 0 *m.* and the side *D, C*, 100, the Angle *B* will be found 14 *d.* 40 *m.* For if you extend the Compasses across from 335 upon the Line of Numbers, to 122 *d.* 0 *m.* (or rather to 58 *d.* 0 *m.* as aforesaid) upon the Line of Sines, and after apply that extent the same way from 100 upon the Line of Numbers, the moveable Point will rest upon 14 *d.* 40 *m.* the measure of the Angle *B* required.

Now having the knowledge of two Angles, the other may be easily discovered, being the Complement of those two to 180, as aforesaid: And the Angles being known, the other side may be also found by the Problem aforegoing.

PROBL. 3.

By the Knowledge of two Sides and the Angle included, to find the other two Angles and the third Side.

If the Angle included be a right Angle, this is the *Proportion:* as the greater side is to the lesser; so is the Tangent of 45 *d.* 0 *m.* to the Tangent of the lesser Angle. And therefore

Extend

Extend the Compasses upon the Line of Numbers downwards from the greater to the less side: then if you apply that extent upon the Line of Tangents the same way from 45 d. *the movable Point will fall upon the Tangent of the lesser Angle.*

Example; In the Rectangle Triangle, *A*; *B*; *D*; of the Diagram aforegoing, the side *A*; *B*; being 230, and the side *A. D*, 143. 72; the Angle *B* will be found 32 *d.* 0 *m.* For, if you extend the Compasses downwards upon the Line of Numbers from 230 to 143. 72, that extent being applied the same way from 45 *d.* at the top of the Line of Tangents, will cause the movable Point to fall upon 32 *d.* 0 *m. viz.* the measure of the Angle *B*; whose Complement 58 *d.* 0 *m.* is the measure of the Angle *D*: And now the three Angles being thus discovered, the third side may also be known by the first Problem of this Chapter.

But If the included Angle be Oblique, *viz.* either obtuse or acute, then this is the *Analogy*: As the Sum of the sides known is, to the difference of the same sides: so is the Tangent of the half Sum of the Angles unknown, to the Tangent of half their difference: And therefore

Extend the Compasses upon the Line of Numbers downwards and upright from the Sum of the given sides; unto their difference: then applying that extent upon the Line of Tangents from the half Sum of the Angles unknown; the movable Point will fall upon the Tangent of half their difference; which being added unto the said half Sum; make up the greater, but being deducted from it discovers the lesser of the Angles you look for.

An *Example* of this Problem, when the moity of the Angles opposed exceeds not 45 *d.*

In the Triangle *B*; *C*; *D*; the side D, B, being 271; the side *D*; *C*; 100, and the Angle *D*; 122 *d.* the Angle *B* will be found 14 *d.* 40 *m.* and the Angle *C*; 43 *d.* 20 *m.* For, if you extend the Compasses upon the Mean

Mean Line of Numbers downwards from 371 (the Sum of the ſides known) to 171 (their difference) that extent being applied the ſame way upon the Line of Tangents from 29 *d.* (half the Sum of the Angles *B* and *C*, the movable Point will fall upon 14 *d.* 20 *m.* which being added to 29 *d.* amounts to 43 *d.* 20 *m.* for the Angle *C*; and being ſubſtracted out of them, the remainder is 14 *d.* 40 *m.* For the Angle *B*.

Two other *Examples* of this Problem, when the moity of the Angles oppoſed exceeds 45 *d.*

1. In the ſame Triangle *C*; *B*; *D*; the ſide *C*; *B*; being 335, the ſide *C*; *D*; 100; and the Angle *C*; 43 *d.* 20 *m.* the Angle *D* will be 122 *d.* and the Angle *B* 14 *d.* 40 *m.* For, if you extend the Compaſſes upon the Line of Numbers downwards from 435 (the Sum of the ſides known) to 235 (their difference) that extent being applied upon the Line of Tangents backwards (*viz.* upwards) from 68 *d.* 20 *m.* (the half Sum of the Angles *D* and *B* required) the movable Point will fall upon 53 *d.* 40 *m.* which being added to 68 *d.* 20 *m.* their Sum is 122 *d.* 0 *m.* *viz.* the meaſure of the Angle *D*; and being deducted out of the ſame 68 *d.* 20 *m.* the remainder is 14 *d.* 40 *m.* the Angle *B*.

2. The ſide *B*; *C*, being 335, the ſide *B*; *D*; 271; and the Angle *B* 14 *d.* 40 *m.* I demand the Angles *D* and *C*: the Sum of the ſides *B*; *C*; and *B*; *D*; is 606, their difference is 64, and the Angle *C* being 14 *d.* 40 *m.* the Sum of the Angles oppoſed and unknown is 165 *d.* 20 *m.* and half that is 82 *d.* 40 *m.* Now to ſatisfie this demand, I extend the Compaſſes upon the Line of Numbers downwards from 606 to 64: then, becauſe if I apply that extent upon the Line of Tangents backwards (*viz.* upwards, as before) from 82 *d.* 40 *m.* the movable Point will fall as far beyond the top of that Line, as the Term I look for is ſituate on this ſide, I apply that extent, down-

downwards from 45 *d.* 0 *m.* causing the movable Point also to fall upon the same Line: that done, and the movable Point remaining there fixed, I close the Compasses till the other Point may rest upon 82 *d.* 40 *m.* And having the Compasses so extended, if applying that extent downwards, I set one of the Points at 45 *d.* the other will reach to 39 *d.* 20 *m.* which being added to 82 *d.* 40 *m.* amounts to 122 *d.* *viz.* the Angle *D*: but being deducted out of 82 *d.* 40 *m.* the remainder is 43 *d.* 20 *m.* *viz.* the measure of the Angle *C*.

And in these three Cases having discovered the three Angles, the other side may be likewise found by the first Problem of this Chapter: Observe also that these two last *Examples* will not admit of *Cross-work*: and therefore are Exceptions to the General Rule delivered in the end of the same Problem.

PROBL. 4.

The three Sides being known, to find the Perpendicular, and the three Angles.

THe greatest side being assigned for the Base, upon which the Perpendicular shall be supposed to fall, find the Sum and the difference of the other sides: that done, the *Proportion* will be this; As the Base is to the Sum of the other sides, so is the difference of the other sides to a fourth Number which being deducted out of the Base, the Perpendicular will fall in the middle of that which remains; Andth erefore

Extend the Compasses upon the Line of Numbers from the parts of the Base unto the Sum of the parts of the other side: this done, and that extent applyed the same way from

from the difference of the other sides, will cause the moveable Point to fall upon a fourth Number, which if you substract out of the intire Base, the Perpendicular will fall in the middle of the remainder.

Example, in the Triangle *E, F, G*, the side *E, F*, being 13, the side *F, G*, 11, and the Base *E, G*, 20, I demand the Point of the Base, where the Perpendicular ought to fall, and then the three Angles of the same Triangle: The Sum of the sides is 24, and their difference is 2: I extend therefore the Compasses upon the Line of Numbers from 20 to 24: that done, in this *Example* (because by the third *Corollary* of the first Problem of the third Chapter, the Numbers 20 and 2 are both represented at the same Point) you may observe (without any farther search) the movable Point to discover the parts of the Segment *E, C, viz.* 2. 4, which being deducted out of 20, there remains 17, 6, whose half is 8, 8, which are the parts of the Base comprehended betwixt *C* and *A*, or betwixt *A* and *G*: I conclude therefore that *A* is the Point of the Base where the Perpendicular ought to fall. Now in the Triangle *A,F,G*, the sides *A, G*, and *G, F*, being known, as also the Angle *F, A, G*, (which is a right Angle by the 10. *Def.* of the 1. *El.* of *Eucl.*) the Angles *G*, and *F*, as also the Perpendicular *F, A*, may be found by the 1 and 2 Probl. of this Chapter. In like manner in the Triangle *E, F, A*, the sides *E, A*, and *E, F*, as also the Angle *E, A, F*, being known, the Angles *E*, and *F*, may be found by the 2. Probl.

of

of this Chapter. And laſtly, if you add the Angle *E, F, A*, and *A, F, G*, together, their aggregate will make up the Angle *E, F, G*: And ſo by the knowledge of the three ſides have you all the parts of that Triangle throughly reſolved.

PROBL. V.

The three Sides being known, to find the Area, or Superficial Content.

FRom the half Sum of the three ſines deduct each ſide, to the end you may diſcover the difference betwixt the ſaid half Sum and each ſide: that done, the *Proportions* will be as followeth:

1. *As* 1 *is to the firſt difference; ſo is the ſecond difference to a fourth Number.*

2. *As* 1 *is to that fourth Number, ſo is the third difference to a ſixth Number.*

3. *As* 1 *is to that ſixth Number, ſo is the half Sum to an eighth Number, whoſe Square-Root is the Area required.*

Example; The three ſides of the foreſaid Triangle *E, F, G*, being 20, 13, and 11, their Sum is 44, half thereof is 22, and the differences betwixt each ſide and that half are 2, 9, and 11: The operation being thus prepared (becauſe the Number required is a Square-Root) I extend the Compaſſes upon the Mean Line of Numbers upwards from 1 to 2: then that extent being applied the ſame way from 9 (in the firſt part of that Line) the movable Point will fall upon 18 the fourth Number: this done, and the movable Point remaining there fixed, cloſe the Compaſſes till the other Point fall again upon 13: for that extent being applied from 11, will cauſe the

the movable Point to fall upon 198, the sixth Number: again, the movable Point remaining there fixed, as before, open the Compasses till the other Point may yet again fall upon 1, and may intercept between the Legs the distance betwixt 1, and 198: for that done, if you apply the same extent (in the first part of the same Line) from 22, the movable Point will fall upon 4356, whose Square-Root (by the 12. Probl. of the last Chapter) will appear at the same Point upon the Great Line of Numbers to be 66, which is also the *Area* required.

2. Of Spherical Rectangle Triangles.

PROBL. 6.

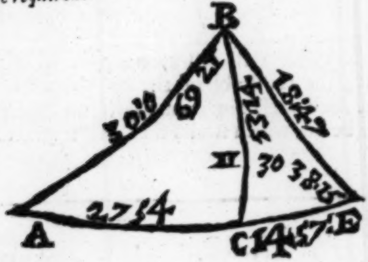
The two Sides being given, to find the Base.

IN Spherical Rectangle Triangles, the side which subtends the right Angle, is called the *Base*, which to find by the knowledge of the other sides, use this *Analogy* following:

As the *Radius* or Sine of 90 *d.* is to the Sine of the Complement (otherwise called the Co-sine) of one of the sides: so is the Co-sine of the other side to the Co-sine of the *Base*: And therefore

Extend

Extend the Compasses downwards uppon the Line of Sines from 90 d. *to the Co-Sine of one of the sides: then applying that extent the same way from the Co-Sine of the other side, the movable Point will rest upon the Co-sine of the Base required.*



Example. In the Triangle *A, B, C,* the side *A, C* being 27 *d.* 54 *m,* and the side *C, B,* 11 *d,* 30 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.* for if you extend the Compasses downwards from 90 *d.* to 62 *d.* 6 *m.* (the Complement of 27 *d.* 54 *m.* and after apply that extent the same way from 78 *d.* 30 *m.* (the Complement of 11 *d.* 30 *m.*) the movable Point will fall upon 60 *d.* being the Complement of 30 *d.* the *Base* required.

PROBL. 7.

The two Sides being known, to find either of the Oblique Angles.

As the Sine of the side next the Angle required is to the *Radius:* so is the Tangent of the opposite side to the Tangent of the same Angle; And therefore

1. When

1. When the side opposed to the Angle required exceeds not 45 d. *Extend the Compasses upon the Line of Sines from the Sine of the side adjacent to the Angle required, to 90* d. *then that extent being applied the same way upon the Line of Tangents, from the Tangent of the side opposed to the required Angle, the movable Point will fall upon the Tangent of the same required Angle.*

1. *Example*, In the said Triangle *A,B,C*, the side *A,C*, being 27 *d.* 54 *m.* and the side *C,B*, 11 *d.* 30 *m.* I demand the Angle *A.* Extend the Compasses upon the Line of Sines from 27 *d.* 54 *m.* to 90 *d.* then that extent being applied the same way upon the Line of Tangents from 11 *d.* 30 *m.* the movable Point will rest upon 23 *d.* 30 *m.* the Angle *A* required.

Or otherwise thus: Extend the Compasses across from 27 *d.* 54 *m.* upon the Line of Sines to 11 *d.* 30 *m.* upon the Line of Tangents: then applying that extent the same way from 90 *d.* upon the Line of Sines, the movable Point will fall upon the Line of Tangents at a Point representing 23 *d.* 30 *m.* as before. And *note*, that in this case the Term required will always fall out to be less then 45 *d.*

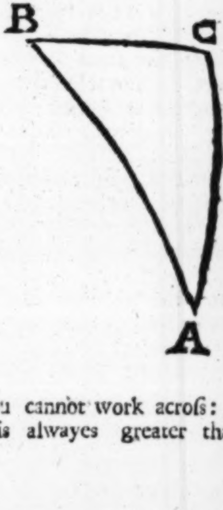
2. *Example*, To know the Angle *B.*: Extend the Compasses upon the Line of Sines from 11 *d.* 30 *m.* to 90 *d.* then (because that extent being applied upon the Line of Tangents the same way from 27 *d.* 54 *m.* will cause the movable Point to fall as far beyond the top of that Line, as the Term required is situate on this side) apply the same extent backwards upon the Line of Tangents from 45 *d.* causing the movable Point to fall also upon the same Line: for, that done, and the movable Point remaining fixed at the Point where it falls, close the Compasses till the other Point may fall upon 27 *d.* 54 *m.* And at last that extent being applied outright upon the Line of Tangents from 45 *degr.* will cause the moveable Point to rest upon 69 *d.* 21 *m.* the Angle *B* required. Or otherwise: Extend the Compasses a-

cross

croſs from 11 *d.* 30 *m.* upon the Line of Sines to 27 *d.* 54 *m.* upon the Line of Tangents: then if you apply that extent backwards from 90 *d.* upon the Line of Sines, the movable Point will fall upon the Line of Tangents at a Point repreſenting 69 *d.* 21 *m.* as before. And here the required Angle is alwayes greater than 45 *d.*

2. When the ſide oppoſed to the Angle required exceeds 45 *d. Extend the Compaſſes upon the Line of Sines from the Sine of the ſide adjacent to the Angle required, to 90 d. That done, if you apply that extent backwards upon the Line of Tangents from the Tangent of the ſide oppoſed to the ſaid required Angle, the movable Point will fall upon the Tangent of the ſame Angle.*

Example, In the *Diagram* annexed, the ſide *A, C*, being 61 *d.* 53 *m.* and *B, C*, 54 *d.* 28 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, the Compaſſes being extended upon the Line of Sines from 61 d. 53 *m.* to 90 *d.* and that extent applied backwards upon the Line of Tangents from 54 *d.* 28 *m.* the movable Point will fall upon 57 *d.* 47 *m.* the Angle *A* required. And here *obſerve* 1. that in *Examples* of this kind you cannot work acroſs: 2. The Angle here found is alwayes greater than 45 *d.*



PROBL. 8.

PROBL. 8.

The Base and one of the Oblique Angles being given, to find the other Oblique Angle.

AS the *Radius* to the Co-sine of the *Base*; so is the Tangent of the Angle known to the Cotangent of the Angle required: And therefore

1. When the Angle given exceeds not 45 *d.* *Ex-end the Compasses upon the Line of Sines from* 90 d. *to he Co-sine of the Base: then if you apply that extent the ame way upon the Line of Tangents from the Tangent of e Angle given; the movable Point will fall upon the o-tangent of the required Angle.*

Example, In the *Diagram* of the sixth Probl. the ase *A, B,* being 30 *d.* and the Angle *A* 23 *d.* 30 *m.* e Angle *B* will be found 69 *d.* 21 *m.* For, if the ompasses be extended upon the Line of Sines from o *d.* to 60 *d.* (the Complement of the Base) and at extent applied the same way upon the Line of angents from 23 *d.* 30 *m.* the movable Point will ft upon 20 *d.* 39 *m.* whose Complement (found al- at the same Point) is 69 *d.* 21. *m.* the Angle *B* re- ired. Or otherwise by cross-work, thus: Extend e Compasses from 90 *d.* upon the Line of Sines to 3 *d.* 30 *m.* upon the Line of Tangents: then that tent being applied the same way from 60 *d.* upon e Line of Sines, the movable Point will fall upon e Line of Tangents at the Point representing 20 *d.* 9 *m.* as before. And here *observe*, that (in this se) the Angle you look for is alwayes less than 5 *d.*

2. When

2. When the Angle given is greater than 45 *d. Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Base: this done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the Angle given, the movable Point will fall upon the Co-tangent of the Angle required.*

1. *Example,* In the *Diagram* of the sixth Probl. *B, A,* being 30 *d.* and the Angle *B* 69 *d.* 21 *m.* the Angle *A* will be found 23 *d.* 30 *m.* For if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* and that extent applied backwards upon the Line of Tangents from 69 *d.* 21 *m,* the movable Point will fall upon 66 *d.* 30 *m.* the Complement of 23 *d.* 30 *m.* the Angle *A* required. And in this case you cannot use cross-work, and the last Term found upon the Rule is alwayes greater than 45 *d.* but the Term required less.

2. *Example,* In the *Diagram* produced in the last Probl. *B, A.* being 74 *d.* 6 *m.* and the Angle *B* 66 *d.* 30 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, if you extend the Compasses upon the Line of Sines from 90 *d.* to 15 *d.* 54 *m.* and then (because that extent being applied backwards, as before, upon the Line of Tangents from 66 *d.* 30 *m.* will cause the movable Point to fall beyond that Line) if you proceed as you were directed in the second *Example* of the said last Probl. at last the movable Point will rest upon 32 *d.* 13 *m.* the Complement of the Angle *A* required. Or otherwise by cross-work: Extend the Compasses from 90 *d.* upon the Line of Sines to 66 *d.* 30 *m.* upon the Line of Tangents: This done, if you apply that extent backwards from 15 *d.* 54 *m.* upon the Line of Sines, the movable Point will rest upon the Line of Tangents at the Point representing 32 *d.* 13 *m.* as before. And (in this case) the last Term found upon the Rule is alwayes less than 45 *d.* but the Term required greater.

PROBL.

PROBL. 9.

The Base and one of the Oblique Angles being known, to find the Side adjacent to the same Angle.

As the *Radius* is to the Co-sine of the Angle known ; so is the Tangent of the Base to the Tangent of the side required : And therefore,

1. When the Base is less than 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known : then applying that extent the same way upon the Line of Tangents from the Tangent of the Base, the movable Point will fall upon the Tangent of the side required.*

So in the *Diagram* of the sixt Problem, *B*, *A*, being 30 *d.* and *A* 23 *d.* 30 *m.* the side *A*, *C*, (whether you work outright or across) will be found 27 *d.* 54 *m.* And in this case the Term required is alwayes lesser than 45 *d.*

2. When the Base exceeds 45 *d.* *Extend the compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known, as before : that done, if you apply the same extent upon the Line of Tangents backwards from the Tangent of the Base, the movable Point will rest upon the Tangent of the side required.*

So in the *Diagram* produced in the seventh Problem, *B*, *A*, being 74 *d.* 6 *m.* and the Angle *A* 57 *d.* 17 *m.* the side *A. C.* will be found 61 *d.* 53 *m.* And in this case you cannot work across, and the side to be found will be always greater than 45 *d.*

Now if in applying the extent of the Compasses from the Tangent of the Base, the movable Point falls

falls beyond the Line, work as you were before directed in the second *Example* of the seventh Problem aforegoing, and so shall you also in that case discover the side you look for, which will then alwayes happen to be less than 45 *d*.

PROBL. 10.

The Base and one of the Oblique Angles being known, to find the Side oppo-sed to the same Angle.

AS the *Radius* is to the Sine of the *Base*, so is the Sine of the Angle known to the Sine of the Side required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. *to the Sine of the Base: For, that extent being applyed the same way from the Sine of the given Angle will cause the movable Point to fall upon the Sine of the side required.*

Example, In the *Diagram* of the sixth Problem; to know the side *B, C*, extend the Compasses upon the Line of Sines from 90 *d*. to 30 *d*. then if you apply that extent the same way from 23 *d*. 30 *m*. the movable Point will fall upon 11 *d*. 30 *m*. the side required.

PROBL. 11.

PROBL. II.

One of the Sides and the Oblique Angle next unto it being known, to find the Base.

AS the Co-sine of the Angle known is to the *Radius*; so is the Tangent of the side given to the Tangent of the Base: And therefore,

1. When the side given exceeds not 45 *d.* *Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *This done, and that extent applied the same way upon the Line of Tangents from the Tangent of the side given, will cause the movable Point to fall upon the Tangent of the Base.* So in the *Diagram* of the sixth Probl. the Angle *A* being 23 *d.* 30 *m.* and the side *A, C,* 27 *d.* 54 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.* But here, if the moveable Point chance to fall beyond the Line, proceed as you have been before directed in the second *Example* of the 7. Probl. And in that case the Term required will alwayes prove greater than 45 *d.*

2. When the given side exceeds 45 *d.* *Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *then, if you apply that extent upon the Line of Tangents backwards from the Tangent of the side given, the movable Point will fall upon the Tangent of the Base.* So in the *Diagram* of the seventh Probl. the Angle *A* being 57 *d.* 47 *m.* and the side *A, C,* 60 *d.* 53 *m.* the Base *B, A,* will be found 74 *d.* 6 *m.* And here the Term sought for is always greater than 45 *d.*

PROBL. 12.

One of the Sides and the Oblique Angle next unto it, being known, to find the other Side.

AS the *Radius* is to the Sine of the ſide given; ſo is the Tangent of the Angle known to the Tangent of the ſide required: And therefore

1. When the Angle given exceeds not 45 *d.* *Extend the Compaſſes upon the Line of Sines from* 90 d. *unto the Sine of the given ſide: this done, and that extent applied the ſame way upon the Line of Tangents from the Tangent of the Angle known; will cauſe the movable Point to fall upon the Tangent of the ſide required;* So in the *Diagram* of the ſixth Probl. *A, C;* being 27 *d.* 54 *m.* and the Angle *A,* 23 *d.* 30 *m.* the ſide *B; C.* will be found 11 *d.* 30 *m.* And in *Examples* of this kind croſs-work may be uſed, and the Term ſought for is always leſs than 45 *d.*

2. When the Angle given exceeds 45 *d.* *Extend the Compaſſes as before: which done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the given Angle, the movable Point will fall upon the Tangent of the ſide required.* So in the *Diagram* of the ſeventh Probl. *B; C,* being 54 *d.* 28 *m.* and the Angle *B,* 66 *d.* 30 *m.* the ſide *A, C.* will be found 61 *d.* 53 *m.* This *Example* and the like cannot be performed by croſs-work; and here the Term found is alwayes greater than 45 *d.* But if in applying the Compaſſes backwards the movable Point chance to fall beyond the Line, work as you were before directed in the ſecond *Example* of the ſeventh Problem

blem of this Chapter, and then will the Term required be alwayes less than 45 *d.*

PROBL. 13.

One of the Sides and the Oblique Angle next unto it being known, to find the other Oblique Angle.

AS the *Radius* to the Co-sine of the given Side; so is the Sine of the Angle known, to the Co-sine of the Angle required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. *to the Co-sine of the side given: this done, that extent being applied the same way from the Sine of the given Angle; will reach to the Co-sine of the Angle required.* So in the *Diagram* of the sixth Problem *A; C;* being 27 *d.* 54 *m.* and the Angle *A* 25 *d.* 30 *m.* the Angle *B* will be found 69 *d,* 21 *m.*

PROBL. 14.

One of the Sides and the Angle opposed unto it being known, to find the Base.

AS the Sine of the Angle given is to the Sine of the side given: so is the *Radius* to the Sine of the Base: And therefore

Extend the Compasses from the Sine of the Angle given

to the Sine of the given side: then if you apply that extent from 90 d. *the movable Point will fall upon the Sine of the Base.* So in the *Diagram* of the sixth Problem, *A*, being 23 *d.* 30 *m.* and the side *B, C*, 11 *d.* 30 *m.* the Base *B, A*, will be found 30 *d.* 0 *m.*

PROBL. 15.

One of the Sides and the Angle opposed unto it being known, to find the other Oblique Angle.

As the Co-sine of the side given is to the Co-sine of the Angle given; so is the *Radius* to the Sine of the Angle required: And therefore,

Extend the Compasses from the Co-sine of the given side, to the Co-sine of the given Angle: this done, that extent being applied the same way from the Radius, *will cause the movable Point to fall upon the Sine of the Angle required.* So in the *Diagram* of the sixth Problem, the side *A, C*, being 27 *d.* 54 *m.* and the Angle *B*, 69 *d.* 21 *m.* the Angle *A*, will be found 23 *d.* 30 *m.*

PROBL. 16.

One of the Sides and the Angle opposed unto it being known, to find the other Side.

As the Tangent of the Angle given is to the Tangent of the side given; so is the *Radius* to the

the Sine of the side required: And therefore,

1. When neither the Angle nor side given exceeds 45 *d. Extend the Compasses downwards upon the Line of Tangents from the Tangent of the Angle given, to the Tangent of the side given: this done, the extent being applied the same way upon the Line of Sines from* 90 *d. will reach to the Sine of the side required.*

So in the *Diagram* of the sixth Problem, the Angle *A* being 23 *d.* 30 *m.* and the side *B, C,* 11 *d.* 30 *m.* the side *A, C,* will be found 27 *d.* 54 *m.*

2. When the Angle and the side given do each of them exceed 45 *d. Extend the Compasses upon the Line of Tangents upwards from the Tangent of the Angle given to the Tangent of the side given, then if you apply that extent backwards upon the Line of Sines from* 90 d. *the movable Point will fall upon the Sine of the side required.*

So in the *Diagram* of the seventh Problem, the Angle *B* being 66 *d.* 30 *m.* and the side *A, C,* 61 *d.* 53 *m.* the side *B, C,* will be found 54 *d.* 28 *m.*

3. When the Angle is greater, and the side less then 45 *d. Extend the Compasses upon the Line of Tangents downwards from* 45 *d. to the Tangent of the Angle given, then if that extent be applied the same way from the Tangent of the given side, the movable Point will fall upon a Point, which upon the Line of Sines represents the Sine of the side required.*

So in the *Diagram* of the sixth Problem, the Angle *B* being 69 *d.* 21 *m.* and the side *A, C,* 27 *d.* 54 *m.* the side *B, C,* will be found 11 *d.* 30 *m.* And here observe, that *Examples* of this kind may likewise be performed by cross-work, the extent of the Compasses being applied backwards: For, having extended the Compasses across from 69 *d.* 21 *m.* upon the Line of Tangents to 90 *d.* upon the Line of Sines, if you apply that extent backwards and across from 27 *d.* 54 *m.* upon the Line of Tangents, the movable Point will fall upon the Sine of 11 *d.* 30 *m.* the side required.

PROBL. 17.

One of the Sides and the Base being known, to find the Angle opposed to the same Side.

AS the Sine of the Base is to the *Radius*; so is the Sine of the side known to the Sine of the Angle required: And therefore,

If you extend the Compasses from the Sine of the Base unto 90 d. *that extent being applied the same way, will reach from the Sine of the great side unto the Sine of the Angle required.* So in the *Diagram* of the sixth Problem, *B, A,* being 30 *d.* and the side *B, C*; 11 *d.* 30 *m.* the Angle *A* will be found 23 *d.* 30 *m.*

PROBL. 18.

One of the Sides and the Base being known, to find the Oblique Angle adjacent unto that Side.

AS the Tangent of the Base is to the Tangent of the given side; so is the *Radius* to the Co-sine of the Angle required: And therefore,

1. When neither the Base nor the side given exceeds 45 *d. the extent from the Tangent of the Base to the Tangent of the side given, being applied the same way, will reach from* 90 d. *to the Cosine of the Angle required.*

So

So in the *Diagram* of the ſixth Problem, the Baſe *B, A,* being 30 *d.* and the ſide *A, C,* 27 *d.* 54 *m.* the Angle *A* will be found 23 *d.* 30 *m.* And in this caſe croſs-work may alſo be uſed, if you apply the Compaſſes the ſame way they were extended.

2. When the Baſe and the ſide given do each of them exceed 45 *d. The extent upwards from the Tangent of the Baſe to the Tangent of the given ſide being applied backwards, will reach from* 90 d. *to the Co-ſine of the Angle required.*

So in the *Diagram* of the ſeventh Problem, the Baſe *B, A,* being 74 *d.* 6 *m.* and the ſide *A, C,* 61 *d.* 53 *m.* the Angle *A* will be found 57 *d.* 47 *m.* Howbeit in this caſe croſs-work hath no place.

3. When the Baſe is greater, and the ſide leſs than 45 *d. Work as you were taught in the third Rule of the ſixteenth Problem aforegoing.*

PROBL. 19.

One of the Sides and the Baſe being known, to find the other Side.

As the Co-ſine of the ſide given is to the *Radius*; ſo is the Co-ſine of the Baſe to the Co-ſine of the ſide required: And therefore,

The extent from the Co-ſine of the ſide given to 90 d. *being applied the ſame way, will reach from the Co-ſine of the Baſe, to the Co-ſine of the ſide required.*

So in the *Diagram* of the ſixth Problem the Baſe *B, A,* being 30 *d.* and the ſide *A, C,* 27 *d.* 54 *m.* the ſide *B, C,* will be found 11 *d.* 30 *m.*

PROBL. 20:

The two Oblique Angles being known, to find the Base.

AS the Tangent of one of the Angles is to the Cotangent of the other Angle; ſo is the *Radius* to the Co-ſine of the Baſe: And therefore,

1. When one of the Angles given, and the Complement of the other are each of them leſs than 45 *d. The extent from the Tangent of the Angle leſs than* 45 d. *unto the Co-tangent of the other, will reach from* 90 d. *to the Co-ſine of the Baſe.* So in the *Diagram* of the ſixth Problem the Angle *A* being 23 *d.* 30 *m.* and the Angle *B* 69 *d.* 21 *m.* the Baſe *B, A,* will be found 30 *d.* And here croſs-work may likewiſe be uſed.

2. When one of the Angles is greater, and the Complement of the other leſs than 45 *d. Proceed as you have been taught in the third Rule of the* 16. *Problem foregoing.*

PROBL. 21.

The two Oblique Angles being known, to find either of the Sides.

AS the Sine of one of the Angles is to the Coſine of the other Angle: ſo is the *Radius* to the Coſine of the ſide oppoſite to the Angle, whoſe Co-ſine was taken: And therefore,

The

The extent from the Sine of one of the Angles given, to the Co-sine of the other, being applied the same way, will reach from 90 d. to the Co-sine of the side opposed to the Angle, whose Co-sine was taken.

So in the *Diagram* of the sixth Problem, the Angle *A* being 23 *d.* 30 *m.* and the Angle *B* 69 *d.* 21 *m* the side *A, C,* will be found 27 *d.* 54 *m.*

3. Of Spherical Oblique Angle Triangles.

PROBL. 22.

Two Angles and a Side opposed to one of them being known, to find the Side opposed to the other.

AS the Sine of the Angle subtended by the side known is to the Sine of the same side; so is the Sine of the Angle subtended by the side required, to the Sine of that side: And therefore,

The extent from the Sine of the Angle opposed to the side known, unto the Sine of the same side, being applied the same way from the Sine of the Angle opposed to the side required, will reach to the Sine of the side so required.

So in the *Diagram* of the sixth Problem, the Angle *E,* being 38 *d.* 15 *m.* the side *B, A,* 30 *d.* and the Angle *A* 23 *d.* 30 *m.* the side *B, E,* will be found 18 *d.* 47 *m.*

PROBL. 23.

Two Sides and the Angle opposed to one of them being known, to find the Angle opposed to the other Side.

AS the Sine of the ſide ſubtending the Angle known is to the Sine of the ſame Angle; ſo is the Sine of the ſide ſubtending the Angle required, to the Sine of that Angle: And therefore,

The extent from the Sine of the ſide ſubtending the Angle known, to the Sine of the ſame Angle, being applied the ſame way, will reach from the Sine of the ſide ſubtending the Angle required, to the Sine of that Angle.

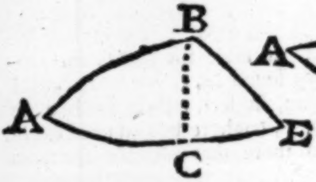
So in the *Diagram* of the ſixth Problem, *B, A,* being 30 *d.* the Angle E 38 *d.* 15 *m.* and the ſide *B. E,* 18 *d.* 47 *m.* the Angle *A* will be found 23 *d.* 30 *m.*

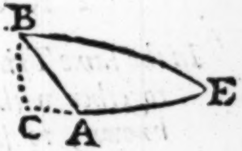
The ſtudious Reader hath by this time (I preſume) ſo well acquainted himſelf with the turnings and windings of this Inſtrument, *that in the reſolution of moſt of the enſuing Problems, it will (I conceive) be only neceſſary to produce the bare* Analogy, *without annexing either Rule or Example as heretofore, and to refer the proper application thereof, to his farther induſtry and diſcretion.*

PROBL.

PROBL. 24.

In any of the Triangles annexed, the Sides A, B, *and* A, E, *together with the Angle* A, *being known, to find the Side* B, E,





IN an Oblique Angle Triangle, when the Terms propounded are two ſides and one Angle, or two Angles and one ſide, and yet the Term required undiſcoverable by the two laſt premiſed Problems, you are to convert ſuch a Triangle into two Rectangle Triangles, by ſuppoſing a Perpendicular to be let fall from any one of the Angles upon his oppoſite ſide, in ſuch ſort that two of the Terms propounded may in one of thoſe Rectangle Triangles ſtill remain given and intire; for by this means all the other parts of ſuch a Triangle thus converted, may be readily diſcovered by the *Analogies* of Rectangle Triangles: And the Perpendicular thus imagined, will fall within the Triangle, when the Angles adjacent to the ſide upon

upon which it falls, are of one and the same kind that is, both acute, or both obtuse; but otherwise without the Triangle, when those Angles are of differing kinds, *viz.* the one acute and the other obtuse, as plainly appears by the Triangles annexed, in which (having the sides *A, B*, and *A, E.* as also the Angle *A* propounded) to find the side *B, E*, use these *Analogies* following:

1. As the *Radius* is to the Co-sine of *A*; so is the Tangent of *A, B*, to the Tangent of *A C*,

2. As the Co-sine of *A, C*, to the Co-sine of *C, E*; so is the Co-sine of *A, B*, to the Co-sine of *B, E*.

And here observe, that to come to the knowledge of *C, E*, in cases that resemble the first of the *Diagrams* annexed, having found *A, C*, you are to deduct it out of *A, E*; again, in such cases as are like the second *Diagram*, *A, E*, ought to be deducted out of *A, C*; and lastly in those that resemble the third *Diagram*, *A, C*, and *A, E*, are to be added together.

PROBL. 25.

In the same Triangles, A,B, *and* A, E, *together with the Angle* A, *being known, to find either of the other Angles, and namely (for Example) the Angle* E.

1, AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A, B*, to the Tangent of *A, C*.

2. As the Sine of *C, E*, to the Sine of *A, C*; so is the Tangent of *A*, to the Tangent of *E*.

PROBL.

PROBL. 26.

A, B, *and* B, E, *together with* A, *being known, to find* A, E.

1 AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A, B,* to the Tangent of *A, C.*

2. As the Co-sine of *A, B,* to the Co-sine of *B, E*; so is the Co-sine of *A, C,* to the Co-sine of *C, E.*

PROBL. 27.

A, B, *and* B, E, *together with* A, *being known, to find* B.

1. AS the *Radius* to the Co-sine of *A, B*; so is the Tangent of *A,* to the Co-tangent of *A, B, C.*

2. As the Tangent of *B, E,* to the Tangent of *A, B*; so is the Co-sine of *A, B, C,* to the Co-sine of *C, B, E.*

PROBL. 28.

A, *and* B, *together with* A, B, *being known, to find either of the other Sides, and namely (for Example) the Side* B, E.

1. As

1. AS the *Radius* to the Co-sine of *A, B*; so is the Tangent of *A*, to the Co-tangent of *A, B, C.*

2. As the Co-sine of *C, B, E*; to the Co-sine of *A, B, C*; so is the Tangent of *A, B*, to the Tangent of *B, E.*

PROBL. 29.

A, *and* B, *together with* A, B, *being known, to find* E.

1. AS the *Radius* to the Co-sine of *A, B*; so is the Tangent of *A*, to the Co-tangent of *A, B, C.*

2. As the Sine of *A, B, C*, to the Sine of *C*; *B*; *E*; so is the Co-sine of *A*, to the Co-sine of *E.*

PROBL. 30.

A, *and* E, *together with* A, B, *being known, to find* A, E.

1. AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A, B*, to the Tangent of *A, C.*

2. As the Tangent of *E*, to the Tangent of *A*; so is the Sine of *A, C*, to the Sine of *C, E.*

PROBL.

PROBL. 31.

A, *and* E, *together with* A, B, *being known, to find* B.

1. AS the *Radius* to the Co-sine of *A, B:* so is the Tangent of *A*, to the Co-tangent of *A, B, C.*

2. As the Co-sine of *A*, to the Co-sine of *E:* so is the Sine of *A, B, C*, to the Sine of *C, B, E.*

PROBL. 32.

Three Sides being known, to find any of the Angles.

ADd the three sides together, then from the half Sum thereof substract the side opposite to the Angle required: this done, the *Proportions* will be as followeth:

1. *As the* Radius *to the Sine of one of the sides including the Angle required: so is the Sine of the other side including the same Angle to a fourth Sine.*

2. *As that fourth Sine is to the Sine of the half Sum of the side: so is the Sine of the difference betwixt that half Sum, and the side opposed to the Angle required, to a seventh Sine, betwixt which and 90* d. *(at the end of the Line of Sines) if you with your Compasses discover the half distance, that Point shall represent unto you an Ark, whose Complement being doubled is the Angle you look for.*

So in the *Diagram* of the 6. Problem the side *A, B*, being

B, being 30 *d.* the ſide *B*, *E*, 18 *d.* 47 *m.* and the ſide *A*, *E*, 42 *d.* 51 *m.* I demand the Angle *B*: The Sum of the Sides is 91 *d.* 38 *m.* half that Sum is 45 *d.* 49 *m.* The ſide *A. E*, being ſubſtracted out of that half, there remains 2 *d.* 58 *m.* And therefore to diſcover the Angle *B*, proceed thus:

Extend the Compaſſes upon the Line of Sines from 90 *d.* unto 30 *d.* then applying that extent the ſame way, and upon the ſame Line from 18 *d.* 47 *m.* the movable Point will fall upon 9 *d.* 16 *m.* Again, that Point remaining there fixed, extend the Compaſſes ſo far that their other Point may reſt upon 45 *d.* 49 *m.* this done, and that extent applied the ſame way from 2 *d.* 58 *m.* will cauſe the movable Point at laſt to fall upon 13 *d.* 20 *m.* whoſe half diſtance towards 90 *d.* will happen upon a Point repreſenting 28 *d.* 42 *m.* whoſe Complement (*viz.* 60 *d.* 18. *m.*) being doubled, amounts to 122 *d.* 36 *m.* the quantity of the Angle *B* required.

PROBL. 33.

The three Angles being known, to find any of the Sides.

IF in ſtead of the greateſt Angle, you take his Complement to 180 *d.* the Angles convert themſelves into ſides, and the ſides into Angles, and then (by conſequent) the operation will be the ſame with that of the laſt *Problem.*

4. Of

4. Of divers other Geometrical Figures.

PRobl. 34. *The Diameter of a Circle being known, to find the Circumference.*

The extent upon the Line of Numbers from 1 to the Diameter, will reach from 3.142 to the Circumference.

Probl. 35. *To find the Superficial Contnt.*

The extent from 1 to the Diameter being twice repeated from .7854, will reach to the Content O, otherwise thus: The extent upon the Great Line of Numbers, from 1 to the Diameter, will reach upon the Mean Line of Numbers from .7854 to the Content: Or yet thus; the Extent upon the Great Line of Numbers from 1 to .7854 will reach upon the Mean Line of Numbers from the Diameter to the Content. And in this manner may divers of the insuing Problems be diversified, which (as before) I refer to the discretion of the Practioner.

Probl. 36. *To find the side of the Square, which may be inscribed within the same Circle.*

The extent from 1 to .7071 will reach from the Diameter to the side of the Square required.

Probl. 37. *Having the Circumference to find the Diameter.*

The extent from 1 to .3183 will reach from the Circumference to the Diameter.

Probl. 38. *To find the Superficial Content.*

The extent from 1 to the Circumference being twice repeated from .07958, will reach to the Content. Or, &c.

Probl.

Probl. 39. *To find the side of the Square, which may be inscribed within it.*

The extent from 1 to the Circumference, will reach from 2251 to the side of the Square required.

Probl. 40. *Having the Content of a Circle, to find the Diameter.*

The extent from 1 to 1. 273 will reach from the Content to another number, whose Square Root is the Diameter required.

Probl. 41. *To find the Circumference.*

The extent from 1 to 12. 57 will reach from the Content to another Number, whose Square Root is the Circumference required.

Probl. 42. *To find the side of the Square equal unto it.*

Bxtract the Square Root thereof by the 12. Probl. of the last Chapter, and you have your desire.

Probl. 43. *The breadth of a long Square being given in Inch-measure, and the length in Foot-measure, to find the Content in Feet.*

The extent from 12 to the breadth in Inches, will reach from the length in Feet to the Content in Feet. Or, *vice versa*, the extent from 12 to the length in Feet, will reach from the breadth in Inches to the Content in Feet.

Probl. 44. *The breadth and length of a long Square being given in Foot-measure to find the Content thereof in Yards.*

The extent from 9 to the breadth, will reach from the length to the Content in Yards. Or, *&c.*

Probl. 45. *To find the Content in single Perches.*

The extent from 16. 5 to the breadth, will reach from the length to the Content in single Perches Or, *&c.*

Probl. 46. *To find the Content in Square Perches; otherwise (in* Architecture*) called* Poles.

The extent from 272. 25 to the breadth, will reach from the length to the Content in Poles. Or, *&c.*

Probl.

Probl. 47. *The breadth and length of a long Square being given in Perches, to find the Content in Acres.*

The extent from 160 to the breadth, will reach from the length to the Content in Acres. Or, &c.

Probl. 48. *The breadth and depth of a Square Rectangle solid, being given in Inch-measure, and the length in Foot-measure to find the Content thereof in Feet.*

The extent from 12 to the breadth or depth in Inches, being twice repeated from the length in Feet, will reach to the Content in Feet. Or, &c.

Probl. 49. *The breadth and depth of a Rectangle solid (not just square) being known) in Inch-measure, and the length in Foot-measure to find the Content in Feet.*

Find (by the tenth *Problem* of the last Chapter) the Mean Proportional betwixt the breadth and the depth; for then, the extent from 12 to that Mean Proportional, being twice repeated from the length in Feet, will reach to the Content in Feet.

Probl. 50. *The breadth and depth of a Rectangle solid (not just square) being known in Foot-measure, to find the Base or Superficies at the end thereof.*

The extent from 1 to the breadth, will reach from the depth to the *Base* required.

Probl. 51. *The Base and length of a Rectangle solid being known in Foot-measure, to find the Content in Feet.*

The extent from 1 to the Base, will reach from the length to the Content.

Probl. 52. *Having the Diameter of a Cylinder, to find the Base.*

The Base of a Cylinder being a perfect Circle, this *Problem* may be resolved by the 35 aforegoing.

Probl. 53. *The Base and length of a Cylinder being known, to find the Content.*

The extent from 1 to the Base, will reach from the length to the Content.

Probl.

Probl. 54. *Having the* Axis *of a Sphere, to find the Superficial Content.*

The extent from 1 to the *Axis*, being twice repeated from 3.142, will reach to the Superficial Content required. Or, &c.

Probl. 55. *To find the solid Content.*

The extent from 1 to the *Axis*, being thrice repeated from .5238, will reach to the solid Content required. Or, &c.

CAP. VI.

The Use of the Rule of Proportion in Astronomy.

PROBL. 1.

By the Sun's Shadow, to find his height.

THe extent upon the Mean Line of Numbers, from the length of the Rules Shadow to the height thereof (held Perpendicular to the Horizon) will reach upon the Line of Tangents from 45 *d.* to the Sun's height required.

Probl. 2. *The Sun's greatest Declination, together with his distance from the next Equinoctial Point being known, to find his present Declination.*

As the *Radius* to the Sine of the Sun's distance from the next Equinoctial Point; so is the Sine of the

the Sun's greatest Declination to the Sine of the Declination required.

Probl. 3. *To find the Right Ascention.*

As the *Radius* to the Tangent of his distance. *&c.* so is the Co-sine of his greatest Declination to the Tangent of his Right Ascension.

Probl. 4. *The Sun's greatest Declination, together with his present Declination, being known, to find his Right Ascension.*

As the Tangent of his greatest Declination to the *Radius*, so is the Tangent of his present Declination to the Sine of his Right Ascention.

Probl. 5. *The Elevation of the Pole, together with the Sun's Declination being known, to find how long the Sun riseth or setteth before or after the hour of six.*

As the Co-tangent of the Elevation is to the *Radius*; so is the Tangent of the Sun's Declination to the Sine of the *Ascentional* Difference between the hour of six, and the Sun's rising or setting.

Probl. 6. *To find the Sun's Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Declination; so is the *Radius* to the Sine of the Amplitude.

Probl. 7. *The Elevation of the Pole, the Sun's greatest Declination, and his distance from the next Equinoctial Point being known to find the Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Sun's distance; so is the Sine of the Sun's greatest Declination to the Amplitude required.

Probl. 8. *When the Sun is in the Equinoctial, by knowing the Elevation of the Pole, to find the Sun's height at any time assigned.*

As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Sun's distance from six a Clock to the Sine of the height required.

Probl. 9. *The Elevation of the Pole, and the Declination of the Sine being known, to find the Sun's height at the hour of six.*

As the *Radius* to the Sine of the Latitude; so is the Sine of the Declination to the Sine of the height required.

Probl. 10. *To find the Sun's height at any time assigned.*

1. As the Radius to the Co-tangent of the Elevation, so is the Sine of the Sun's distance from six, to the Tangent of an Ark, which being subtracted out of the Sun's distance from the Pole, I say again.

2. As the Co-sine of the Ark found is to the Co-sine of the residue of the Sun's distance from the Pole; so is the Sine of the Elevation to the Sine of the height required.

Probl. 11. *To find the time when the Sun will be due East and West.*

As the Tangent of the Elevation to the *Radius*; so is the Tangent of the Declination to the Co-sine of the hour from the Meridian.

Probl. 12. *To find the Sun's height, when he cometh to be due East and West.*

As the Sine of the Elevation to the *Radius*; so is the Sine of the Declination to the height required.

Probl. 13. *To find the Sun's Azimuth at the hour of six.*

As the Co-sine of the Elevation is to the Co-tangent of the Declination; so is the *Radius* to the Tangent of the Azimuth from the North part of the Meridian.

Probl. 14. *The Complement of Elevation, the Sun's distance from the Pole, and the Complement of the Sun's height being known, to find the Azimuth.*

Having added the three given Terms together, find the difference betwixt their half Sum and the Sun's distance from the Pole: this done, the *Proportion* will be as followeth:

1. As the *Radius* to the Co-sine of the Elevation; so is the Co-sine of the height to a fourth Sine:

2. As

2. As that fourth Sine is to the Sine of the half Sum; so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Ark, whose Complement being doubled is the *Azimuth* you look for.

Probl. 15. *To find the hour of the Day.*

Having added the three given Terms together, as before, find the difference betwixt their half Sum and the Complement of the Sun's height; this done, the *Proportions* will be these:

1. As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Sun's distance from the Pole to a fourth Sine.

2. As that fourth Sine is to the Sine of the half Sum: so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Ark, whose Complement being doubled and converted into Time, will produce the hour required.

CAP. VII.

The Use of the Rule of Proportion in Dialling.

PROBL. 1. *To make a direct Polar Dial.*

HAving assigned a Line drawn in the middle of the Plane for the Meridian, and another Line

Line drawn parallel unto it for ſome other hour, which may be deſcribed upon the Plane: I ſay,

1. As the Tangent of that hour is to the *Radius*; ſo is the diſtance of that Hour-line from the Meridian to the height of the Stile.

2. As the *Radius* is to the height of the Stile; ſo is the Tangent of any other hour, to the diſtance of the ſame hour from the Subſtile.

Probl. 2. *A Meridian Dial.*

Having drawn a Line repreſenting part of the *Axis* of the World towards a proper ſide of the Plane, (according to his ſituation either Eaſtward or Weſtward) aſſigned that Line for the hour of ſix, the *Proportion* will fall out to be as in the former *Problem*; for,

1. As the Tangent of any hours diſtance from ſix is to the *Radius*; ſo is the diſtance of the hour upon the Plane from the Hour-line of ſix, to the height of the Stile.

2. As the *Radius* is to the height of the Stile; ſo is the Tangent of any other hours diſtance from ſix to the diſtance of the ſame hour from the Subſtile.

Probl. 3. *An Horizontal Dial.*

As the *Radius* to the Tangent of the hour given; ſo is the Sine of the Elevation to the Tangent of the Hour-line from the Meridian.

Probl. 4. *A Vertical Dial.*

As the *Radius* to the Tangent of the hour: ſo is the Co-ſine of the Elevation of the Tangent of the Hour-line from the Meridian.

Probl. 5. *A Vertical Inclining Dial.*

Having found out the Elevation of the Pole above the Plane, according to its inclination, the Proportion will be this:

As the *Radius* to the Tangent of the Hour: ſo is the Sine of the Elevation above the Plane, to the Tangent of the Hour-line from the Meridian.

Probl.

Probl. 6. *A Vertical Declining Dial.*

1. As the *Radius* to the Co-tangent of the Elevation : ſo is the Sine of the Declination to the Tangent of the Subſtile diſtance from the Meridian of the Place.

2. As the *Radius* to the Co-ſine of the Declination : ſo is the Co-ſine of the Elevation to the Sine of the Stile's height above the Subſtile.

3. As the Sine of the Elevation is to the *Radius* : ſo is the Tangent of the Declination to the Tangent of the Inclination of the Meridian of the Plane to the Meridian of the Place.

4. As the *Radius* to the Sine of the Stile's height above the Subſtile : ſo is the Tangent of the Angle at the Pole comprehended between the hour given and the Meridian of the Plane, to the Tangent of the Hour-lines diſtance from the Subſtile.

Probl. 7. *A Meridian Inclining Dial.*

1. As the *Radius* to the Tangent of the Elevation : ſo is the Sine of the Inclination to the Tangent of the Subſtile's diſtance from the Meridian.

2. As the *Radius* is to the Sine of the Elevation : ſo is the Co-ſine of the Inclination to the Sine of the Stile's height above the Subſtile.

3. As the Co-ſine of the Elevation is to the *Radius* : ſo is the Tangent of the Inclination, to the Tangent of the Inclination of Meridians.

4. As the *Radius* is to the Sine of the Stile's height above the Subſtile : ſo is the Tangent of the Angle at the Pole, to the Tangent of the Hour-lines diſtance from the Subſtile.

Probl. 8. *A Polar Declining Dial.*

1. As the *Radius* to the Sine of the Declination : ſo is the Co-ſine of the Elevation to the Co-ſine of the Ark comprehended between the Horizon and the Subſtile.

2. As the *Radius* to the Tangent of the Declination : ſo is the Sine of the Elevation to the Tan-

gent of the Inclination of Meridians, which being converted into time, sheweth how many hours the Substile ought to be placed from the Hour-line of 11.

3. As the *Radius* is to the Tangent of the hours distance from the Substile: so are the parts of the height of the Stile, to the distance of the Substile from the Hour-line required, measured by a Scale of like parts.

Probl. 9. *A Declining Inclining Dial.*

1. As the *Radius* to the Tangent of Inclination to the Horizon: so is the Co-sine of Declination to the Tangent of the Ark of the Meridian of the Place intercepted between the Horizon and the Plane, which being compared with the Elevation of the Pole, the distance of the Pole from the Plane may be thereby readily discovered.

2. As the *Radius* is to the Sine of Declination from the Vertical: so is the Sine of Inclination to the Horizon, to the Co-sine of the Inclination to the Meridian.

3. As the *Radius* is to the Co-sine of Inclination to the Horizon: so is the Cotangent of Declination to the Tangent of the Ark of the Plane intercepted between the Horizon and the Meridian of the Place.

4. As the *Radius* is to the Sine of the Inclination to the Meridian: so is the Tangent of the Pole's distance fcom the Plane, to the Tangent of the Substile's distance from the Meridian.

5. As the *Radius* is to the Pole's distance from the Plane: so is the Sine of the Inclination to the Meridian, to the Sine of the Stile's height above the Substile.

6. As the Cosine of the Pole's distance from the Plane is to the *Radius*: so is the Cotangent of the Inclination to the Meridian, to the Tangent of the Inclination of Meridians.

7. As

7. As the *Radius* is to the Stiles height above the Subſtile; ſo is the Tangent of the Angle at the Pole, to the Tangent of the Hour-line's diſtance from the Subſtile.

CAP. VIII.

The Uſe of the Rule of Proportion in Geography.

Probl. 1. *Two Places being propounded, which differ only in Latitude, to find their Diſtance.*

1. WHen the two places are ſituate under the ſame Meridian, and upon the ſame ſide of the Equinoctial; *Subſtract the leſſer Latitude out of the greater; that done, the remainder is the diſtance required.*

2. When one of the places propounded is ſituate upon this ſide the Equinoctial, and the other upon that, and yet both under the ſame Meridian, as before: *Add the two Latitudes together; this done, their Sum is the diſtance required.*

Probl. 2. *Two places, which differ only in Longitude, being propounded, to know their diſtance.*

1. When the Places are both of them ſituate under the Equinoctial: *Subtract the leſſer Longitude out of the greater: this done, the remainder is the diſtance required.*

2. When the Places are ſituate under ſome Parallel betwixt the Equinoctial and one of the Poles: *Then, as the Radius is to the Coſine of the common Latitude given: ſo is the Sine of half the difference of Longitude to the Sine of half the diſtance.*

Probl. 3. *Two places being given, which differ both in Longitude and Latitude, to find their diſtance.*

1. When one of the Places is ſituate under the Equinoctial, and the other towards one of the Poles: Then, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-ſine of the Latitude given, to the Coſine of the diſtance required.*

2. When both Places are without the Equinoctial, and towards one of the Poles: Then, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-tangent of the leſſer Latitude to the Tangent of another Ark*, which being ſubſtracted out of the Complement of the leſſer Latitude, retain the Ark thereof remaining; and ſay again, *As the Co-ſine of the Ark found is to the Co-ſine of the Ark remaining: ſo is the Sine of the leſſer Latitude to the Co-ſine of the diſtance required.*

3. When both Places are without the Equinoctial, and one of them ſituate towards the North Pole, and the other towards the South: ſay thus, *As the Radius is to the Co-ſine of the difference of Longitude: ſo is the Co-tangent of one of the Latitudes, to the Tangent of another Ark*, which being ſubſtracted out of the other Latitude, and 90 *d.* added together: ſay again, *As the Co-ſine of the Ark found is to the Co-ſine of the Ark remaining: ſo is the Sine of the Latitude firſt taken, to the Co-ſine of the diſtance required.*

CAP.

CAP. IX.

The Use of the Rule of Proportion in Navigation.

Probl. 1. *The Latitudes of two Places being known, to find the Meridional Difference.*

1. WHen one of the Places is situate under the Equinoctial, and the other without: *The Degrees and Decimal Minutes found upon the Scale of Equal Parts at the Point; where that other Latitude is represented upon the Scale of Latitudes, are the Meridional difference required.*

2. When one of the Places have Southerly, and the other Northerly Latitude: *Extend the Compasses upon the Line of Latitude, from the beginning of that Line to the lesser Latitude: that done, if you apply that extent upon the same Line, and the same way from the greater Latitude, the movable Point will discover upon the Line of equal Parts, the Meridional difference desired.*

3. When both Places have Northerly or Southerly Latitude: *Extend the Compasses upon the Line of Latitudes from one of the Latitudes to the other: this*

done, if you apply that extent from the beginning of the Line, the movable Point will shew you upon the Scale of Equal Parts the Meridional difference you look for.

Probl. 2. *The Latitudes of two places together with their difference of Longitude being known, to find the Rumbe directing from the one to the other.*

As the Meridional difference is to the difference of Longitude: so is the *Radius* to the Tangent of the Rumbe: And therefore,

The extent upon the Mean Line of Numbers from the Meridional difference to the difference of Longitude, will reach upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe.*

And *note* here, that in this Problem and the like, you may make use of the double Scale, placed upon the last Line of the Rule of Proportion, at the end of the Scale of Inches: *viz.* (if need be) for the more speedy reduction of the Sexagenary Minutes of the Longitude into Decimals, & *contra*: to the end you may by that means the more readily work by them upon the Mean Line of Numbers.

Probl. 3. *By both Latitudes and Rumbe to find the distance upon the Rumbe.*

As the Co-sine of the Rumbe to the true difference of Latitudes: so is the *Radius* to the distance required: And therefore,

Extend the Compasses across from the Co-sine of the Rumbe (found upon the Line of Sines) to the true difference of Latitudes (found upon the Mean Line of Numbers) this done, if you apply that extent the same way and across from 90 d. *upon the Line of Sines, the movable Point will shew you upon the Mean Line of Numbers (in Degrees and Decimal Minutes) the distance required.*

Probl. 4. *By both Latitudes and Rumbe, to find the difference of Longitude.*

As the *Radius* to the Tangent of the Rumbe: so is the Meridional difference of the Latitudes to the

the difference of Longitude required : And therefore

The extent upon the Line of Tangents from 45 d. to the Tangent of the Rumbe, will reach upon the Mean Line of Numbers from the Meridional difference of the Latitudes to the difference of Longitude required.

Probl. 5. *By both Latitudes and distance to find the Rumbe.*

As the distance is to the true difference of Latitudes : so is the *Radius* to the Co-sine of the Rumbe: And therefore,

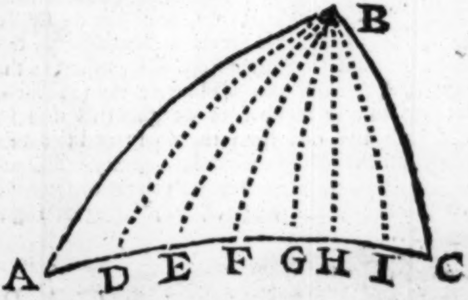
The extent upon the Mean Line of Numbers, from the distance to the difference of Latitudes, will reach upon the Line of Sines from 90 d. to the Co-sine of the Rumbe.

Probl. 6. *By one Latitude, distance, and Rumbe, to find the other Latitude.*

As the *Radius* to the Co-sine of the Rumbe : so is the distance to the true difference of Latitudes : And therefore,

The extent upon the Line of Sines from 90 d. to the Co-sine of the Rumbe, will reach upon the Mean Line of Numbers, from the distance to the true difference of Latitudes.

Probl. 7. *The Latitudes and difference of Longitude of two places being known, to sail by the great Circle from the one to the other.*



 In

In the Triangle *A, B, C*, let *A* repreſent *S. Chriſtophers*, *C*, the *Lizard*, *B*, the North Pole, *A, B*, the Complement of the Latitude of *S. Chriſtophers, viz.* 74 *d.* 30 *m. B, C.* the Complement of the Latitude of the *Lizard*, 40 *d.* 0 *m.* and *A, B, C*, the difference of Longitude, 68 *d.* 30 *m.* Now therefore to ſteer a courſe from *A* to *C* alongſt the Ark *A, C*, proceed thus:

1. By the 24 and 25 *Problems* of the fifth Chapter find the ſide *A, C*, as alſo the Angles *A*, and *C*.

2. By the 22 of the ſame Chapter find the Perpendicular *B, J*, cutting the ſide *A, C*, at Right Angles.

3. By the 8 of the ſame diſcover the Angle *A, B, I*, and by the 9 the ſide *A, I*.

4. Leſſening the Angle *A, B, I*, two, five, or ten Degrees, as you ſhall ſee cauſe, (for *Example*, by the Angle *A, B, d*,) by the knowledge of the Angle *d, B, I*, and of the ſide *B, I*, find by the 11, 12, and 13 *Problems* of the ſame fifth Chapter, the Baſe *B, d*, the ſide *d. I*, and the Angle *B, d, I*; and ſo proceeding to do the like at the Points *e, f, g*, and *h*, you may thereby diſcover the ſeveral diſtances betwixt Point and Point, the ſeveral Latitudes at thoſe Points, and the ſeveral Angles according to which you are to direct your Courſe: For at firſt, from *A* you are to ſteer according to the Angle *B, A, I*, until you ſhall have ſailed ſo many Leagues as anſwer to the diſtance betwixt *A* and *d*: and then from *d*, according to the Angle *B, d, I*, untill you ſhall arrive at the Point *e*, according to the number of Leagues that *d* and *e* are diſtant the one from the other: and ſo conſequently of the reſt in their order, until you ſhall attain the Point *I*, from whence you are to ſteer full Weſt towards *c*, the Angle *B, I, C*; being a Right Angle, *&c.*

CAP. X.

The Use of the Rule of Proportion in the Gaging of Vessels.

Probl. 1. *The true Content of a Solid Measure being known, to find the Gage Point of the same Measure.*

THe Gage Point of a solid Measure is the Diameter of a Circle, whose Superficial Content is equal to the solid Content of the same Measure so the solid Content of a Wine-gallon (according to *Winchester* measure) being 231 Cube-inches, if you conceive a Circle to contain so many Inches, you shall find (by the fortieth *Problem* of the fifth Chapter) the *Diameter* thereof to be 17. 15: For,

As 1 *is to* 1.273 : *so is* 231 *to* 294. 1, *whose Square root* (by the twelfth *Problem* of the same Chapter) *is* 17. 15, *the Gage-point of Wine-measure.*

Thus likewise may you easily discover the Gage-point of Ale-measure, an Ale-gallon (as it hath been of late discovered) containing 288 Cube-inches: For,

As 1 *is to* 1. 273 : *so is* 288 *to* 366. 7, *whose Square-root is* 19.15, *the Gage-point of Ale-measure.*

And (indeed) 288 Cube-inches seem to be the most probable Content of an Ale-gallon, being the sixth part of 1728, which is the Number of Cube-inches contained in a Cube-foot. For so (according to that account) a Cube-foot contains just six Gallons, and the Gage-point of Ale-measure (by reason of the soil and waste) exceeds that of Wine-measure just two Inches.

After the same manner also may you discover the Gage-point of any Forreign measure whatsoever, and afterwards by that means come to the knowledge of the true Content of their Vessel, according to the Measures used amongst them, as will plainly appear by that which shall hereafter be taught for the discovery of the Contents of Wine and Beer-vessel according to the English Measures.

Now from that which is abovesaid doth necessarily follow this Corollary: *When the Diameter of a Cylinder in Inches is equal to the Gage-point of any Measure (given likewise in Inches) every Inch in the length thereof contains one Integer of the same Measure:* So in a Cylinder having 17.15 Inches Diameter, every Inch in the length thereof contains one intire Wine-gallon: and in another having 19.15 Inches Diameter, every Inch thereof contains one Ale-gallon, *&c.*

Probl. 2. *In a Wine or Beer-vessel, the Diameters at the Head and Bongue being known, to find the required Diameter.*

Extend the Compasses upon the *Line of Inches* from the Diameter at the Head, to the Diameter at the Bongue: then applying that extent from the beginning of the same Line, and observing there the difference betwixt the two Diameters, (one of the Points remaining still fixed at the beginning of that Line) close the Compasses till the other Point may fall upon so many parts of the *Gage-line*, as the difference between the two Diameters, amounts unto in Inches: this done, and that extent applied from the Diameter

Diameter at the Head towards the Diameter at the Bongue, will cause the movable Point to fall upon the *Equated* Diameter you look for.

Example, The Diameter at the Head being 18. 3 Inches, and that at the Bongue 21. 5 Inches, I demand the *Equated* Diameter. First, extending the Compasses upon the Line of Inches from 18. 3 Inches to 21. 5, and then applying that extent from the beginning of the same Line, I find the movable Point to fall upon 3. 2 Inches, *viz.* the true difference of the two Diameters: Now therefore if still keeping one of the Points of the Compasses fixed at the beginning of that Line, I close them till the other Point may fall at 3. 2 upon the *Gage-line*, and after apply that extent from 18. 3 (the Diameter at the Head) the movable Point will at last fall upon 20. 54 Inches, the *Equated* Diameter required. And by this means your Vessel, which before was in part of an Oval form and irregular, is now reduced into a perfect *Cylinder*.

Probl. 2. *The equated Diameter and length of a Wine or Beer-vessel being given in Inches, to find the Content thereof in Wine-measure.*

The extent upon the *Line of Numbers* from 17. 15 (the Gage-point of Wine-measure) to the *Equated* Diameter, being twice repeated from the length, will reach to the Content in Wine-gallons.

Probl. 4. *To find the Content in Ale-measure.*

The extent from 19. 15 (the Gage-point of Ale-measure) to the *Equated* Diameter; being twice repeated from the length, will reach to the Content in Ale-gallons.

Probl. 5. *Having the length and the two Diameters at the Head and Bongue, together with the Equated Diameter and Content of a Vessel, and of which so much and no more of the liquor is drawn: that the Superficies thereof may cut some part of the Head, to find the true quantity of the remainder.*

Deduct

Deduct half the difference of the Diameters at the Head and Bongue, out of the distance intercepted between the Bongue and the Superficies of the Liquor, to the end you may thereby discover where the Liquor within the Vessel cuts the Head, according to which draw a Line with Chalke (or otherwise) upon the Head, then having drawn another Line parallel to the first, and of like distance from the other opposite side of the Head, you have in the middle of the Head betwixt those two Lines a Segment of the Vessel marked out, and likewise two other Segments, the one above and the other below that middle Segment: after this taking the length of one of those Parallels in Inch-measure, the *Equated* Diameter of the Superficies may be thus found out upon the *Rule*:

The extent from the Diameter at the Head to the Equated Diameter of the Vessel, will reach from the length of one of the Parallels to the Equated Diameter of the Superficies.

Then having discovered (by the 2d *Problem* aforegoing) the *Equated* Diameter of those two other *Equated* Diameters, find (by the tenth *Problem* of the fourth Chapter) the Mean Proportional between that third *Equated* Diameter and the distance between the two Parallels: This done, make use of that Mean Proportional, as an *Equated* Diameter of the middle Segment, and then finding by one of the two last *Problems* according to the Question propounded) the Content thereof in Gallons, &c. deduct that Content out of the whole Content of the Vessel: All this performed, *when the Vessel is above half full*, the Content of that middle Segment and half that remainder being added together, is the Content you look for. But, *when the vessel is not half full*, half that remainder is the Content desired.

CAP.

CAP. XI.

The Use of the Rule of Proportion in Military Orders.

Probl. 1. *Any Number of Soldiers being propounded, to order them into a Square Battail of Men.*

FInd (by the twelfth *Problem* aforegoing(the Square-root of the Number given: For, look how much that Root shall happen to be, so many Soldiers ought you to place in Rank, and so many likewise in File, to make a Square Battail of Men.

Example; Let it be required to order 573 Soldiers into a Square Battail of Men: the Square-root of that Number is 23.94: and therefore you are to place 23 in Rank, and as many also in File: For; Fractions are not considerable in Questions that concern, *Military* Orders.

Probl. 2. *Any Number of Souldiers being propounded to order them into a double Battail of Men: viz. which may have twice so many in Rank as in File.*

Find out the Square-root of half the Number given: for that Root is the Number of Soldiers to be placed in File: and so many more ought to be placed

placed in Rank, to make up a double Battail of Men.

Example, 1342 Souldiers being propoueded to be put into that order: I find 26,*&c.* to be the Square-root of 671 (half the Number propounded) and thereupon conclude that 26 ought to be placed in File, and 52 in Rank, to order so many Soldiers into a double Battail of Men.

Probl. 3. *Any Number of Soldiers being given, to order them into a quadraple Battail:* viz. *such as may have four times so many in Rank as in File.*

Here the Square-root of the fourth part of the Number given will shew the Number to be placed in File, and sometimes so many are to be placed in Ranks.

So 2048 Soldiers being offered to be put into that order, 22 are to be placed in File, and 88 in Rank. For, the fourth part of 2048 is 512, whose Square-root is 22, *&c.*

Probl. 4. *Any Number of Soldiers being given, together with their distance in Rank and File, to order them into a Square Battail of Ground.*

Extend the Compasses upon the Mean Line of Numbers from the distance in File to the distance in Rank: this done, and that extent applied the same way, and upon the same Line from the Number of Soldiers propounded, will cause the moveable Point to fall upon a fourth Number, whose Square-root appearing at the same Point upon the Great Line of Numbers is the whole Number of Men to be placed in File: by which if you divide the Number of Soldiers, the Quotient will shew the Number of Men to be placed in Rank.

Example, 2500 Men are propounded to be ordered into a Square Battail of Ground, in such sort that their distance in File being seven foot, and their distance in Rank three foot, the Ground whereupon they stand may be a just Square. To resolve this Question, extend the Compasses upon the Mean Line of

of Numbers downwards from 7 to 3: then (because the fourth Number to be found in all likelihood will consist of four Figures) if you apply that extent the same way from 2500 in the first part of the same Line, the movable Point will fall upon the fourth Number you look for, where also you may observe 32, *&c.* upon the second part of the Great Line of Numbers, which are the Number of Men to be placed in File; again, if letting that Point of the Compasses remain fixed there, you close them till the other Point may reach cross-wise to 1 at the beginning of the first part of the said Great Line of Numbers, that extent being applied the same way (*viz.* downwards and across) from 2500 upon the same Great Line, the movable Point will fall near 78, *&c.* which are the Number of Soldiers to be placed in Rank.

Probl. 5. *Any Number of Soldiers being propounded, to order them in Rank and File according to the reason of any two Numbers given.*

This Problem is resolved much after the same manner that the last was: For,

As the Proportional Number given for the File is to that given for the Rank: so is the Number of Souldiers to a fourth Number, whose Root is the Number of Men to be placed in Rank, by which if you divide the whole, the Quotient is the Number to be placed in File.

So if 2500 Soldiers were to be marshalled in such order, that the Number of Men to be placed in File might bear such proportion to the Number of Men to be placed in Rank, as 5 bears to 12: I say then, as 5 is to 12, so is 2500 to another Number, whose Root is 77, *&c. viz.* the Number of Men to be placed in Rank, by which if the same 2500 be divided, the Quotient will be 32, *&c.* the Number of Men to be placed in File.

CAP.

CAP. XII.

The Uſe of the Rule of Proportion in Queſtions that concern Intereſt and Annuities.

Probl. 1. *A Sum of Money being forborn for a certain time, to find how much it will be augmented at the expiration of the ſame time, accounting Intereſt upon Intereſt, according to a certain rate propounded.*

THe extent upon the Line of Numbers from 100 *l.* to the aggregate of 100 *l.* and the rate added together, being repeated the ſame way from the Sum given, ſo many times as there are years in the Queſtion, will at laſt cauſe the movable Point to fall upon the Principal increaſed with the Intereſt, according to the forbearance and rate propounded.

Example, I deſire to know how much 273 *l.* being forborn for five years will be increaſed at the expiration of thoſe years according to Intereſt upon

on Intereſt, and the rate of 8 *l. per centum*: Extend the Compaſſes upon the great Line of Numbers from 100 to 108: This done, if that extent be repeated five times from 273, the movable Point will at laſt fall upon 402. 1 (*viz.* 402 *l.* 2 *s.*) the Principal augmented with the Intereſt for the forbearance of thoſe five years.

Probl. 2. *A Sum of Money being due at a time to come, to find what it is worth in ready Money.*

This is the *Inverſe* of the laſt: for here, if you apply that extent backwards from the Number propounded, ſo many times as there are years in the Queſtion, you ſhall have your deſire.

Example, 402 *l.* 2 *s.* being due at the end of five years yet to come, I deſire to know how much that Sum is worth in ready Money according to the rate of 8 *l. per centum*: Extend the Compaſſes from 100 to 108, as before: And then, if you apply that extent five times downwards from 402. 1, the movable Point will at laſt fall upon 273 *l.* the value of 402. 1, in ready Money.

Probl. 3. *A yearly Rent or Annuity being forborn a certain Number of years, to find what the Arrearages thereof will amount unto according to any rate propounded.*

Firſt diſcover the principal that anſwers to the Rent or Annuity in queſtion, then find unto what Sum that Principal will be augmented (according to the given rate) at the end of the Term propounded: This done, if you ſubſtract the ſame Principal out of that Sum, the remainder is the Sum of the Arrearages you look for.

Example, A Rent or Annuity of 12 *l. per annum* being forborn 16 years, what will the Arrearages thereof amount unto, they being conceived to increaſe (as they grow due) after the rate of 8 *l. per centum.* Here firſt, to find the Principal that anſwers to 12 *l.* ſay thus: If 8 *l.* hath 100 l. for his Principal,

Principal, what ought 12 *l.* to have for his? the answer will be (by the fourth *Problem* of the fourth Chapter) 150 *l.* Having thus discovered the Principal of 12 *l.* *viz.* 150 *l.* I find (by the first *Problem* of this Chapter) that the same 150 *l.* being forborn 16 years will amount (after the rate of 8 *l.* *per centum*) to 513. 9, that is 513 *l.* 18 *s.* Now therefore if I deduct 150 *l.* (the Correspondent Principal to the Annuity given) out of 513 *l.* 18 *s.* the remainder *viz.* 363 *l.* 18 *s.* is the Sum of the Arearages required.

Probl. 4. *A yearly Rent or Annuity being propounded, to find what it is worth in ready money.*

First, find what the Arrearages thereof amount unto at the end of the Term propounded, and then what those Arrearages are worth in ready money, which shall likewise be the required price or value of the Rent or Annuity propounded.

Example, What may a man which is desirous to lay out his money after the rate of 8 *l.* *per centum*, afford to give for a Lease of 12 *l.* *per annum* that hath yet 16 years in being? I find (by the last *Problem*) that the Arrearages of 12 *l.* *per annum*, being forborn 16 years, amount then unto 363 *l.* 18 *s.* or 363. 9, and I find likewise (by the second *Problem* aforegoing) that the same 363 *l*, 18 *s.* is worth in present money 106. 2, or (which is all one) 106 *l.* 4 *s.* I conclude therefore that the value of the Lease propounded (at the rate of 8 *l.* *per centum*) is 106 *l.* 4 *s.*

Here, when the Term of the Annuity begins not presently, but after certain years to come, find what the Arrearages forborn for all that time are worth in ready money.

So in the *Example* last premised, if the Annuity of 16 years were not to begin till after the expiration of 5 years; in this case you are to enquire what the Arrearages (*viz.* 363 *l.* 18 *s.* being forborn 21 years, are worth in ready money, which you shall likewise find

find (by the second *Problem* before cited) to be 72. 3, which being reduced is 72 *l.* 6 *s.* the value of the Leafe required.

Probl. 5. *A Sum of Money being propounded, to find what Annuity (to continue any Number of Years, and according to any rate given) that Sum will buy.*

Take any Annuity at pleafure, then find the value of that Annuity in ready money: This done, the *Proportion* will be as followeth:

As the value found is to the Annuity taken; so is the Sum given to the Annuity required.

Example, What Annuity (to continue 16 Years) will 1205 *l.* deferve, fo that the purchafer may gain after the rate of 8 *l. per centum?* Here, firft, I take 12 *l. per annum* to continue 16 years, and find the value thereof in ready money (by the laft *Problem*) to be 106. 2, or 106 *l.* 4 *s.* I fay therefore,

If 106. 2 give 12 *l. per annum.*

What will 1205 *l.* yield? *Facit* 171,4 *per annum*, which being reduced is 171 *l.* 8 *s.* I conclude therefore, that 171 *l.* 8 *s.* is the Annuity (to endure 16 years) which 1205 *l.* doth deferve, after the rate of 8 *l. per centum.*

Deo Laus.

FINIS.

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Wingate's RULE OF PROPORTION IN ARITHMETICK AND GEOMETRY: OR, GUNTER'S LINE

Newly rectified by Mr. *Brown* and Mr. *Atkinson*, Teachers of the Mathematicks.

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Astronomy,	Gaging of Vessel,
Dialling,	Military Orders,
Geography,	Interest and
Navigation,	Annuities.

London, Printed by *R. H.* for *W. Fisher*, *T. Passinger*, *R. Boulter*, and *R. Smith*, 1683.

Thom. Tanner

TO MY

Worthy Friend,

AND ABLE

MATHEMATICIAN,

Mr. *John Collins*

OF

LONDON.

SIR,

NOt long after my Arrival in this City, having divulged the Instrument (whose Uses I explain in this little Treatise) and discoursed of some of the conveniences thereof, I was given to understand by

 divers

divers, that if pains were bestowed upon that Subject, the Labour therein taken might obtain good Reception: This (to say truth) hath given me Encouragement thereof to say somewhat; and (having caused it to see the Light) to shelter it under your Protection: Nevertheless you shall pardon me, for that by presuming to procure unto it from thence Credit and Recommendation I have expressed a willingness to testifie, how much I am,

Your Servant,

Edmond Wingate.

THE

THE PREFACE TO THIS TRANSLATION.

AMongſt the many rare Effects produced by the noble Invention of *Logarithmes*, the projection of the *Rule* of *Proportion* is not the leaſt, which being firſt diſcovered by that Learned and Induſtrious Artiſt *Edm. Gunter* (late Profeſſor of *Aſtronomy* in *Greſham* Colledge, *London*, deceaſed) was by me (in *Anno* 1624.) tranſported into *France*, and there communicated to moſt of the chief-

of being in this form once gained, the Practitioner may then uſe that way of deſcribing it, which ſorts beſt with his own humor.

Having thus acquainted you with the occaſion of publiſhing this Treatiſe, leſt I may now expoſe it to prejudice, give me leave to premiſe theſe few Advertiſements following: Firſt, therefore, it is deſired, that he, who intends to read this Book with profit, ſhould have a proper *Genius* and Phanſie for the *Mathematicks*, not only ready to conceive *Mathematical* Notions; but likewiſe able to wreſtle with them, and apt to take pleaſure in them: For, *De quolibet ligno non fit Mercurius.* Again, it is expected he ſhould be aforehand furniſhed with competent knowledge in thoſe Sciences, *viz.* 1. In *Arithmetick* he ought to be acquainted with the Nature of Numbers, whole and broken, abſolute and relative; with Numeration,

tion, Addition, Subſtraction, Multiplication, Diviſion, the Rule of Three, direct and inverſe; with the Nature and Extraction of Roots, Square and Cube; And with the right uſe of *Logarithms*: 2. In *Geometry*, to be verſt in the Doctrine of Triangles, plain and ſpherical, and (in ſome competent meaſure) to know their nature, together with the way and reaſon of their dimenſion; As alſo the dimenſion of other Geometrical Figures: 3. In *Aſtronomy*, *Dialling*, and *Geography*, to underſtand that the Problems which concern them, are reſolved by the particular application of the Doctrine of Spherical Triangles to thoſe ſeveral Sciences: 4. In *Navigation*, to be indifferently well read in ſuch Authors as have explained that Art, and to be able therein alſo to make uſe of the Doctrine of Triangles: With the knowledge of theſe things (I ſay) and

and the like he ought to be (in ſome reaſonable ſort) ſupplied, that intends to make a right and complete uſe of this Treatiſe: For, none (I preſume) will expect to find an intire Body of the *Mathematicks* in this ſmall Bulk, which is only intended for an *Enchiridion* or Manuel of ſuch Mathematical Rules and *Analogies*, as may moſt properly ſerve for the reſolution of Problems, which may be wrought upon this *Inſtrument*: And therefore I wholly refer the *Reader* for demonſtrations and larger explanations of the matters in this Book contained, to the further ſcrutiny of other Authors; Not doubting but that (upon due peruſal hereof) he will find as much inſerted, as ſhall be thought neceſſary to diſcover the manifold and exquiſite uſe of the ſame *Inſtrument*. But here I would not be miſtaken, as if I did totally exclude all others, who are

not

not prepared with ſuch an Univerſal Knowledge in the *Mathematicks*, from having any capacity at all of underſtanding this Book; For, if he be only in part acquainted with ſome of the abovementioned Learning, he may be able to make uſe of this *Inſtrument* according to that degree of Knowledge which he hath therein; For Example, if he only know *Multiplication* and *Diviſion*, this Treatiſe will inſtruct him how to *multiply* and *divide* upon the *Rule*, and ſo in like ſort of the reſt: Howbeit (as I ſaid before) if he intend to have an intire underſtanding of the uſes of this *Inſtrument*, he muſt be alſo furniſhed with an intire knowledge of all the *Mathematicks*; becauſe it is ſubſervient to every Branch of thoſe Sciences: And then the conveniency thereof will have ſuch Latitude, that it will not be confined to thoſe uſes only promiſed in the Title of this Book, but

likewiſe

likewise (by the variety of Rules and Examples therein found) may be readily and fitly applied to other Arts and Professions not there remembred; As namely, in *Fortification*, the Ingenier may here be taught how to find the Sides of his *Polygonical* Figures, the *Lines* of Fortification according to the Rules of that Art, the *quantity* of Trenches and Ramparts, how to order and estimate the labour and work of Pioners, and the like. The *Surveyor* also may here furnish himself with divers expeditious dispatches, for the taking of distances, the summing up of Plots, being first divided into Triangles, the distribution of Fields or Lordships to several Persons, the cutting off any part of a Triangle or Plot according to any quantity propounded, *&c.* The like may be said of *Musick*, *Architecture*, the *Prospectives*, *Gunnery*, *&c.* The *Goldsmith* also, and *Mint-Master*

ſter may here learn how to temper their Allegations: The *Merchant* and *Tradeſman*, how to reſolve queſtions of Partnerſhip, and to caſt up the value of their Commodities: The *Juſtice* of *Peace* and *High Conſtable*, how to rate a Town, Hundred, or County, *&c.* All which and much more muſt be wholly left to the diſcretion of thoſe, that will take the pains to underſtand the uſe of the ſaid *Inſtrument*; which (I perſwade my ſelf) no man (affecting the *Mathematicks*) will think much to undergo, conſidering the benefit he may reap thereby, and the delight he may take therein; For, by help thereof, and of a pair of Compaſſes, only ſix Inches long, he may reſolve with requiſite exactneſs any Propoſition in the Arts and Sciences above remembred (which comes within the bounds of ordinary practice) without the help of Pen or Paper, and ſhall thereby alſo perform

perform more in one hour, then otherwiſe (I mean by ordinary *Arithmetick*) he ſhall be able to diſpatch in two whole days.

But it may be objected, if this *Inſtrument* be of ſuch excellent uſe as is here pretended, why hath it not been heretofore of greater eſteem, it being now above twenty years ſince it was firſt invented? This Objection may be anſwered divers ways: 1. It is no eaſie matter to drive men out of their old track, eſpecially when they have entertained an opinion that there can be none better. 2. Again, the uſe thereof in the point of *Numbring* upon the *Rule* (which ought to be accounted the chiefeſt, and indeed the ground of all the reſt) hath not been heretofore (under favour) ſo fully explained, as here you ſhall find it: For, albeit (I confeſs) it were great preſumption in me to aſſume to my ſelf the reputation of

having

having better abilities to deſcribe any of the uſes thereof, then Mr. *Gunter* himſelf had, who firſt invented it; yet this I can aver upon mine own knowledge, that he did forbear to explain the uſe thereof, becauſe he took it for granted none would meddle with it but ſuch only as were already well able to underſtand how to number upon it, having before-hand acquainted themſelves with the manner of *Numbring* upon *Scales*, aud with the nature of *Logarithms*: For, when after my return out of *France*, I importuned him to make a fuller explanation, how to number upon it, to the end the uſe thereof might by that means be made more publick, his anſwer was, *That it could not be expected the Rule ſhould ſpeak*; Intimating thereby, that the Practitioner ſhould (in that point) rely much upon diſcretion, and not altogether depend upon Precepts and Examples. But laſt-

ly, the chiefeſt cauſes why this *In-ſtrument* hath been hitherto obſcu-red and the uſes thereof no better known to the World, are theſe. 1. The Difficulty of deſcribing the Lines thereupon with convenient exactneſs: 2. The trouble of work-ing thereupon by reaſon (ſome-times) of too large an extent of the Compaſſes: 3. The importableneſs thereof, it being requiſite for work-ing upon ſuch a *Rule* (only two foot long) to uſe a pair of Compaſſes of nine Inches: 4. The charge of pur-chaſing ſuch an *Instrument* made of Braſs or Wood; For, none but ſuch have been heretofore uſed. For remedy of the firſt of theſe, I have cauſed the Plate, whereupon this *Inſtrument* is Printed, to be protracted with a great deal of care and circumſpection, ſo that I dare affirm it to be as exactly drawn (for the main and moſt conſiderable Di-viſions thereof) as may be expect-

ed

ed from Art: For the ſecond, having there three ſeveral Lines of *Numbers* by degrees one leſs than another, when the Compaſſes are too little for one, you may uſe another, alſo *Croſs-work* upon the greateſt Line will prevent the too great extenſion of the Compaſſes; ſo that it will be requiſite to uſe with this *Inſtrument* (as it is now contrived) a pair of Compaſſes only ſix Inches long, as I ſaid before; and yet the Diviſions of this (I mean upon the great Line of *Numbers*) are near as large again, as thoſe upon Mr. *Gunter*'s Rule of the like length: The third and fourth impediments may alſo be remedied, if in ſtead of Braſs or Wood you uſe the impreſſion of the ſaid Plate upon Vellum or Imperial Paper, which may either be rolled up and couched in a little Box, or otherwiſe paſted upon a Ruler either flat, to uſe at home, or round, to be carried

in

in a hollow Staff or Cane together with the Compaſſes, which are to be uſed therewith. Alſo divers uſeful conveniences ſhall you meet withall in this Edition of the *Rule*; as namely, a readier way of finding out *Mean-Proportions*, the *Extraction* of Roots by Inſpection only, without aid of Pen or Compaſſes, and the like: For further diſcovery of all which I refer you to the Book it ſelf, hoping that my real intention to advance the Publick Good will procure from the Ingenuous *Reader* a favourable conſtruction of what he ſhall therein find not wilfully miſtaken.

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By *James Atkinson.*

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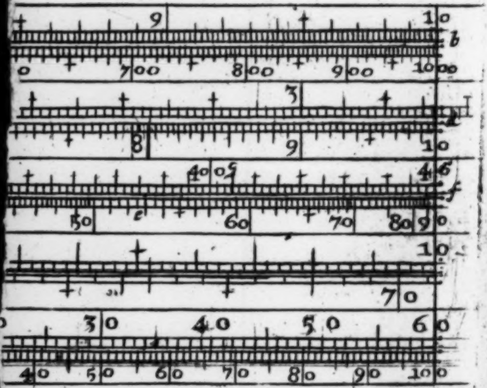
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THE USE OF THE Rule of Proportion in Arithmetique and Geometrie.

CAP. I.

The Description of the Scales projected upon the Rule of Proportion.

U*Pon the five Lines of the Rule of proportion, there are ten several Scales projected, viz. two upon each common or middle Line, the one having the Divisions thereof shooting downwards, the other upwards:* So the first two Scales meet upon the middle or common Line *a, b,* the next two upon the Line *c. d,* &c.

The uppermost or first Scale of the Rule is a single Line of Numbers; first divided into nine unequal parts, called *Primes,* and distinguished by the Figures, 1.2.3.4.5. 6. 7. 8. 9. And then, each of those *Primes*, subdivided into ten other Parts (according to the same Reason) called *Tenths:* And again, each of those *Tenths* subdivided, or at least supposed to be subdivided into ten other Parts, as the length of

the Rule will admit: For *Example*, upon the Sche
of our Rule (hereunto annexed) which is ſuppo
to be about two foot and three inches long betw
the end-lines in the four firſt *Primes* (*viz.* betw
the Figures 1 and 5) each *Tenth* is really ſubdivi
into ten Parts; but in the reſt of the *Primes* (*viz.*
tween the Figure 5, and the end of that Scale) e
Tenth is divided but into five Parts; and there
each of thoſe five Parts ought to be eſteeme
have the value of 2; and the ſaid tenth parts
thoſe *Tenths* are hereafter called *Centeſmes*: Laſt
each of thoſe *Centeſmes* is alſo ſuppoſed to be ſub
vided into ten leſſer Parts, which are hereafter c
ed *Millains*: By all which you may obſerve, t
the longer the Rule is, the more ſmall Diviſion
will admit, and the ſhorter it is, the fewer.

The ſecond Scale is another Line of Numbers thrice
peated: This Scale ſhoots upwards upon the Co
mon Line *a, b*, and being of a leſſer Volume th
the former, muſt in ſome Parts thereof conten
ſelf with leſs Diviſions, *viz.* from the Figure of 5
the end of that Scale the *Tenths* are only divi
into two Parts, and therefore each of thoſe
Parts ought to retain the value of five: All
three Parts of this Scale being taken together, a
hereafter (for diſtinction ſake) called the *Little L*
of Numbrs, and are in their uſe diſtinguiſhed by
firſt, ſecond, and third Part, as they lie in orde
They are alſo of ſingular uſe for the ready diſcov
of the Cube-root, and for the reſolution of other
ceſſary operations, as ſhall be ſhewed hereafter.

The third Scale is the firſt Scale repeated, taking
beginning from the middle of the Rule, and being bro
off at the upper end thereof, is afterwards continued fro
the lower end of the ſame to the place where it firſt beg
This Scale abuts downwards upon the Comm
Line *c, d*; and the firſt and this being taken tog
ther are hereafter called the *Great Line of Numbe*

where

whereof the firſt Scale is called the firſt Part, and this the ſecond.

The fourth Scale is another Line of Numbers twice repeated: This Scale ſhoots upwards upon the Common Line *c, d,* and being intirely taken together, is hereafter called the *Mean Line of Numbers:* It conſiſteth alſo of two Parts, diſtinguiſhed by firſt and ſecond, as they lye in order; and is of neceſſary uſe for the finding of the Square-root, and of mean Proportions, as ſhall appear hereafter.

The fifth Scale is a Line of Tangents; This Scale abuts downwards upon the common Line *e, f,* and doth firſt contain the Artificial Tangents of the Quadrant from 0. *degr.* 35. *min.* to 45. *degr.* at the upper end of that Scale, and ſo if the Rule would permit, ſhould they be continued forward to 89. *degr.* 25. *min.* but becauſe the Diviſions of that Scale being inverted, will fall out to be the ſame with the former, they are to be noted and accounted backwards from 45. *degr.* at the upper end of that Scale to 89. *degr.* 25. *min.* at the lower end of the ſame; each degree thereof being ſubdivided into ſix Parts, and each of thoſe ſix Parts ſuppoſed to contain ten minutes.

The ſixth Scale is a Line of Sines: Upon this Scale ſhooting upwards upon the Common Line *e, f,* are deſcribed the Artificial Sines of the Quadrant from 0. *degr.* 35. *min.* to 90. *degr.* at the upper end of that Scale, each degree (upon our Rule) from 0. *degr.* 35. *min.* to 30. *degr.* being ſubdivided into ſix Parts, each Part repreſenting ten minutes, as thoſe of the Tangents; but from 30. *degr.* to 50. only into four Parts, each Part containing 15 minutes; from 50 to 70, into two Parts, each Part comprehending 30 minutes; from 70 to 85, into eaven degrees; and laſtly, from 85. *degr.* to 90, not divided at all, but ſuppoſed to be divided into five Parts, repreſenting thoſe five laſt degrees of the Quadrant.

The seventh Scale shooting downwards, is the whole Rule divided into 1000 *equal Parts*; It is hereafter called the *Scale of equal Parts*, and is of use for the Construction and Fabrick of the Great Line of *Numbers.*

The eighth Scale shooting upwards, is a Scale of 7 *degr.* 11 *min. of the Quadrant described according to* Mercator *and Mr.* Wright's *Projection*: It is hereafter called the *Scale of Latitudes,* and is to be used together with the *Scale of equal Parts*; and both of them taken together, are usually called the *Meridian Line* and are of excellent use in Navigation, as shall be declared hereafter.

The ninth is the Scale of Inch-measure, viz. two foot thereof divided into 24 inches, and each inch into ten lesser Parts, counted both forwards and backwards, after the usual manner!

The tenth and last Scale consists of three several kinds viz. a Gage Line, a Line of Cords, and a Scale of Foot-measure: The first of these being signed by the Letter *G*, is nothing else but seven inches divided into ten equal Parts, and those subdivided into ten lesser Parts, and is hereafter to be used for the ready discovery of the *equated* Diameter (and so by consequent of the Content) of any Wine, Beer, or Oyl Vessel: The next marked by the Letter *C*, is an ordinary Line of *Cords*, already sufficiently known, and of frequent use amongst Artists; the third and last, marked by the Letter *F*, is the *Scale of Foot-measure* being nothing else but a foot first divided into ten Parts, and those subdivided into ten lesser Parts, and so (by consequent) the whole foot supposed to be thereby divided into 1000 Parts.

At the end of these two Scales there is another double Scale placed, containing in length three inches *French*, whereof the uppermost shooting downwards, is a Scale divided into 60 Parts, and that shooting upwards into 100 Parts: The use of these

these two Scales is for the ready reduction of Sexagenary minutes to Decimals, and of Decimal minutes to Sexagenaries, as shall appear hereafter.

CAP. II.

The Construction and Fabrick of the Lines described upon the Rule of Proportion.

1. TO *describe the* Line of Numbers, *having prepared a Rule of Silver, Brass, or Wood, (of what length you please) and caused it to be ruled according to the Pattern hereunto annexed, and also a Scale of* 1000 *equal Parts to be drawn, equal in length to your intended* Line of Numbers, *repair to the Table of Logarithms, and therein observing the first four Figures of the Logarithm of* 101, *beside the Index or Characteristick* (viz. 0043) *take with your Compasses the distance from the beginning of the said Scale of equal Parts to the said* 43 *Parts; This done, if you apply that extent of the Compasses upwards, from the beginning of the* Line of Numbers, *which you intend to make, the moveable Point of the Compasses, will fall upon the first* Centesme *of that Line: In like manner by the first four Figures of the Logarithm of* 102, *besides the Index* (viz. 0086) *you may mark the second* Centesme *of the same Line, and so consequently all the rest in their order.*

Example, If it were propounded to make a *Line of Numbers* equal to that of the first Scale, let there be a Scale of equal Parts made, equal in length to

that *Line*, such as the seventh Scale before described happens to be: then extending your Compasses from the beginning of that Scale of equal Parts to 0043, *viz.* to the Point *a*, apply that extent from the beginning of your Intended *Line of Numbers*; For, that done, the moveable Point of the Compasses will fall upon the first *Centesm* of that *Line*, *viz.* at the Point *e*: In like manner, the extent from the beginning of the Scale of equal Parts to 0086, *viz.* to the Point *c*, will mark out upon the intended *Line of Numbers* the Point *b*, representing the second *Centesm* of that *Line*, and so consequently the rest in order.

2. The *Line of Tangents* is framed much after the same manner; For, having before prepared a Scale of equal Parts sutable to that *Line*, (*viz.* consisting of half the length of the whole Line) *Repair unto the Table of Artificial Sines and Tangents, and therein finding the Artificial Tangent of* 0. *degr.* 40. *min. if (rejecting the Characteristick or first Figure thereof) you take off with your Compasses upon your foresaid sutable Scale of equal Parts (as before) the four first Figures of the same Tangent (viz.* 0658*) that extent being applied upwards from the beginning of the* Line of Tangents, *will cause the moveable Point of the Compasses to fall upon the Division, representing* 0. *degr.* 40. *min. In like manner the extent of* 1627 *(the second, third, fourth, and fifth Figures of the Tangent of* 0. *degr.* 50. *min.) will guide to mark out the same* 0. *degr.* 50. *min. upon the same Line:* And so proceeding you may readily describe all the rest, as they follow in order.

3. The *Line of Sines* may be drawn in all Points, as the *Line of Tangents*, if you use the second, third, fourth and fifth Figures of the *Artificial Sines*, as you are before directed to use those of the *Tangents*. And here *note*, that the Line before called the *Mean Line of Numbers*, and these Lines of *Tangents* and *Sines* are all of them framed by one and the same Scale,

Scale, and are alſo hereafter to be uſed together in the reſolution of *Plain Triangles,* the Scale of equal Parts or *Radius*, by which they are made, being in each of them twice repeated.

4. The *Meridian Line* being framed by the ordinary Table of Meridional Degrees, and the making of the *Line of Cords* being obvious to every mean Practitioner in the Mathematicks, I ſhall not need to trouble you with their Conſtruction. The other Scales alſo, which conſiſt of equal Parts, will not need any farther deſcription.

CAP. III.

Numeration upon the Rule of Proportion.

PROBL. I.

A whole Number being given, to find the Point where the ſame is repreſented upon the Line of Numbers.

Find amongſt the Figures, by which the Primes are diſtinguiſhed, the firſt Figure of the Number given, and for the ſecond Figure there of count from the beginning of the Prime, unto which the firſt Figure directs you, ſo many Tenths as that Figure hath Unites; Then for the third Figure count from the laſt Tenth ſo many Centeſms as that

third Figure hath Unites: And so likewise for the fo[urth] Figure count from the last Centesme so many Millains [as] the same fourth Figure hath Unites: This done, you [sh]al last fall upon the Point where the Number propoun[ded] is represented upon the Line of Numbers.

Example, The Number given being 1728, the f[irst] Figure thereof (*viz.* 1.) leads me unto the first *Pri[me]* designed by the Figure 1, within which *Prime* cou[nt]ing seven *Tenths* for the second Figure, and from [the] seventh *Tenth* two *Centesmes*, for the third Fig[ure], and from the second *Centesme* eight *Milains* for [the] fourth Figure; at last I find the Number given to [be] represented upon the first Part of the Great Line [of] Numbers at the Point *h*: So likewise is the Numb[er] 27 found at the Point *k*, the Number 542 at t[he] Point *l*, and 3345 at the Point *m*, &c.

From hence follow these Corollaries:

1. *The Figure which any Number given hath towar[ds] the right hand, besides the first four Figures towards t[he] left hand, are not expressed upon the Rule*: And therefo[re] if the Number given were 172845, it would be lik[e]wise represented at the Point *h*: Howbeit, that u[n]certainty causeth no inconvenience in the use of t[he] Rule, as shall more plainly appear hereafter.

2. *The Figures by which the Primes are distinguish[ed] (in reference to one and the same Number) retain alwa[ys] one and the same value.*

Example, In searching the Number 1728, concei[v]ing the Figure prefixed at the beginning of the fir[st] *Prime* (*viz.* 1.) to have the value of *Thousands*, the F[i]gure prefixed before the second *Prime* (*viz.* 2.) oug[ht] also to be esteemed to have the value of *Thousand[s]* and so of the rest in their order: for, according t[o] the same reason that *h* represents 1728, the Point [o] will represent 2000, the Point *p* 3000, &c.

3. *The Numbers, which have only the simple value [of] Unites as* 1. 2. 3, 4. *&c. and these which after the fir[st] Figure*

Figure have nothing but Cyphers, as 10.100.1000.20.200. 2000. *&c. are all represented at the same Points.*

So 1.10.100.1000.*&c.*may be all represented at the beginning or end of the *Line* 2. 20. 200. 2000. *&c.* at the beginning of the second *Prime:* 3. 30. 300. 3000. *&c.* at the beginning of the third *Prime, &c.*

4. *The Numbers, which being composed of three Figures have a Cypher in the middle, are found betwixt the beginning of the Prime, unto which they belong, and the first Tenth of the same Prime.*

So 405 beginning by the Figure 4, (and therefore to be sought for in the fourth *Prime*) is represented at the Point *o*.

5. *The Numbers, which being composed of four Figures, have now Cyphers in the middle, are represented betwixt the beginning of the Prime, unto which they belong, and the first Centesme of the same Prime:* So 1005 is found at the Point *q*.

6. *When the Line of Numbers is repeated, and for that cause consisteth of several Parts; the first Part thereof is in value a degree less than the second, and the second a degree less than the third,* &c.

So upon the Mean Line of Numbers, if you conceive 10 at the upper end thereof to represent 100, the Figure 1 in the middle (or which is all one, at the beginning of the second Part) will represent 10, and 1 at the lower end of that Line (or which is all one, at the beginning of the first Part) will represent 1: But if 10 at the upper end thereof shall be conceived to bear but the value of 10, the Figure 1 in the middle shall have the value of one, and one at the lower end the value of $\frac{1}{10}$, and 2 the value of $\frac{2}{10}$, *&c.* In like manner, if 10 at the upper end represent 1, the Figure 1 in the middle must represent $\frac{1}{10}$, and 1 at the lower end $\frac{01}{100}$, *&c.*

PROBL. 2.

To find a Fraction or broken Number upon the Line of Numbers.

THe Fractions, which are to be found upon the Line of Numbers, ought always to be Decimals, *viz.* ought always to have for their Denominators the Figure 1, with nothing but Cyphers towards the right hand, such as are $\frac{125}{1000}$ $\frac{25}{100}$ $\frac{5}{10}$ $\frac{75}{100}$ or the like, which may otherwise be written thus, .125.25,5,75, and are equivalent to $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ and $\frac{3}{4}$: And therefore if the Fractions propounded be not Decimals, they ought to be reduced to such: For, that done, they may be discovered in all Points as whole Numbers are found out upon the Line, which may be plainly understood by the *Examples* produced in the sixt *Corollary* of the last Problem.

PROBL. III.

To find a Mixt Number upon the Line of Numbers.

F*Irst find by the first Problem aforegoing the Point representing the whole Parts of the Number given, and then afterwards the Fraction or broken Parts thereof in the Ranks that follow.*

Example, a Line that hath the length of 17 foot and $\frac{28}{100}$ of a foot (which may more conveniently

be

be written thus, 17, 28) being propounded; first, I find the whole Parts thereof, *viz.* 17) represented at the Point *r*, and after counting two Centesms, and then eight Millains, at last I find the Number given to be represented at the Point *h*. In like manner if the Number propounded were 172. 8, or 1. 728, it would be still represented at the same Point.

PROBL. IV.

Any Point of the Line of Numbers being assigned, to find the Figures represented at the same Point.

Take the Figure prefixed at the beginning of the Prime, within which the Point is propounded, for the first of the Figures required; then shall the second Figure required be composed of so many Unites as there are Tenths intercepted betwixt the beginning of the same Prime and the Point given. In like manner shall the third Figure required have so many Unites as there are Centesms comprehended betwixt the last of those Tenths and the said Point: And so likewise shall the fourth Figure consist of so many Unites as there are Millains between the last Centesm and the Point given.

Example, If the Point *h* were propounded, because that Point is situate within the Prime, before which the Figure 1 is prefixed, I take the Figure 1 for the first of those required; and then finding seven Tenths betwixt the beginning of that Prime and the Point given, I set down 7 for the second: And so proceeding and finding two Centesms betwixt the last Tenth and the said Point, I take 2 for the third Figure: And lastly; conceiving eight Millains to be comprehended between the last Cen-

tesine and the Point given, I take 8 for the fourth Figure required: This done, I conclude, that the Figures represented at the Point propounded, are 1728. In like manner the Point *q* being given, I take 1 for the first Figure; but here because I find no Tenths betwixt the beginning of that Prime and the Point given, I write a Cypher in the second place; and there also finding no Centesines, I write also a Cypher in the third place; And then at last finding the Point propounded in the middle of a Centesme (which is supposed to be divided into ten Millains) I annex in the fourth place 5: This done, the Figures represented at the Point given will be found 1005.

PROBL. 5.

An Ark or Angle being propounded to find upon the Rule of Proportion the Point which represents the Tangent of the same Ark or Angle

If the Ark or the measure of the Angle exceeds not 45 *degrees, search the degrees of that Arke or Angle upon the Line of Tangents, mounting upwards from the lower end of that Line towards the upper end of the same.*

So the Tangent of an Ark or Angle, which consists of 15 degrees, is represented at the Point *a*: of 25 degrees at the Point *b*, *&c.*

But if the Ark or measure of the Angle exceeds 45 *degrees, look the degrees thereof, descending downwards from the upper end of the Line towards the lower end of the same:* So the Tangent of 65 degrees is found at the

the Point *b*, of 75 degrees at the Point *a*, *&c.*

And if the Ark or Angle propounded (besides the whole degrees) is also composed of certain minutes, find first the whole degrees, and after that, betwixt the last degree found, and the next that follows, take so many of the Parts which may amount to the minutes given accounting each of the Parts contained betwixt the two degrees for ten minutes: So the Tangent of 22 *degr.* 45 *min.* is found at the Point *d*, and the Tangent of 72 *degr.* 45 *min.* at the Point *a*. And therefore *e converso*, if the Points *d* and *t* were given upon this Line, the degrees and minutes represented by them, would be 22 *degr.* 45 *min.* and 72 *degr.* 45 *min.* &c.

PROBL. 6.

An Ark or Angle being propounded, to find upon the Rule of Proportion the Point, which represents the Sine of the same Ark or Angle.

F*Ind upon the Line of Sines the degrees of the Ark or Angle given, and you have your desire:* So the Sine of the Ark or Angle of 22 *degr.* is represented at the Point *r*.

But if the Ark or Angle given have also minutes annexed, first search the whole degrees given, and then between that degree found and the next that follows, take so many Parts as you have minutes propounded, conceiving the distance betwixt each degree, and the next that follows to comprehend 60 *minutes.*

So the Sine of 22 *degr.* 45 *min.* is found at the Point *u*; of 42 *degr.* 50 *min.* at the Point *q*; of 52 *degr.* 45 *min.* at the Point *c*, *&c.* And therefore here also

alſo *e converſo*, if the Points *u*, *q*, and *e* were aſſigned upon this Line, the degrees and minutes repreſented by them would be 22 *degr.* 45 *min.* 42 *degr.* 50 *min.* and 52 *degr.* 45 *min.* &c.

CAP. IV.

The uſe of the Rule of Proportion in Arithmetick.

IN *Arithmetick* there are three ſeveral ſorts of Proportion; *Arithmetical*, *Geometrical*, and *Muſical*. *Arithmetical*, when divers Numbers being compared together retain amongſt themſelves equal differences, as theſe, 2. 4. 6. 8. *&c.* And this is either *continued*, as in the Numbers before produced, or in theſe, 3. 6. 9. 12. 15, *&c.* which is alſo called *Arithmetical* Progreſſion, or a Rank of Numbers *Arithmetically* proportional; or diſcontinued, as in theſe, 2. 4. 10. 12, or the like. *Geometrical* Proportion is, when divers Numbers being compared together differ amongſt themſelves according to the ſame rate or reaſon, as theſe, 2. 4. 8. 16. *&c.* For here, as 2 is half 4, ſo is 4 half 8, and 8 half 16: this is likewiſe either *continued*, as in thoſe before propounded, or in theſe, 1. 3. 9. 27. 81. *&c.* or the like, which is alſo called *Geometrical* Progreſſion, or a Rank of Numbers *Geometrically* proportional: Or *diſcontinued*, as in theſe, 2. 4. 16. 32; for as 4 is double 2, ſo is 32 double 16, but ſo is not 16 being compared with 4. *Muſical* Proportion is that which doth as it were proceed from both the former, as when three Numbers or Terms being propounded, the firſt bears the ſame Proportion to the third, that the difference betwixt

betwixt the first and the second bears to the difference betwixt the second and third, as in these, 3. 4. 6, for here, as 3 is half 6, so is 1, the difference betwixt 3 and 4, to 2, the difference betwixt 4 and 6. So likewise 2. 3. 6. and 10. 16. 40. are said to be Numbers *Musically* proportional: For, in the first of these two last Examples, as 2 is to 6, so is 1 to 3; And in the others, as 10 is to 40, so is 6 to 24. Thus have I here thought fit briefly to remember the *Reader* of the several kinds of Proportion, which he doth usually find in the Writings of those that treat of *Arithmetick*; to the end that the *Problems* which follow both in *Arithmetick* and *Geometry* may be the better understood.

PROBL. I.

Two Numbers being given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

Extend the Compasses upon the Line of Numbers from one of the Numbers given to the other; this done, if you apply the same extent (upwards or downwards) from either of the Numbers propounded, the moveable Point of the Compasses will fall upon the third proportional required: And so the same extent being applyed the same way from the third, the moveable Point of the Compasses will fall upon the fourth proportional, and from the fourth upon the fifth, &c.

Example, Let it be propounded to find a third proportional to these two Numbers 2 and 4, which may bear the same Proportion to 4, that 4 bears

to

to 2; First, I Extend the Compasses upon the first Part of the Mean Line of Numbers from 2 to 4; this done, if I apply that extent outright from 4 upwards, the moveable Point of the Compasses will fall upon 8 the third Proportional required; and being applied the same way from 8, the movable Point will rest upon 16, the fourth Proportional; and from 16 to 32, the fifth; and from 32 to 64, the sixth Proportional. But now if you would yet continue the Progression farther, and so find the next Proportional to 64 (because the movable Point in that case will fall beyond the Line) apply that extent the same way from 64 in the first Part of that Line; which done, the movable Point of the Compasses will then fall upon 128, the seventh Proportional; and so proceeding farther you may find 256, the eighth 3512, the ninth, *&c.*

Contrariwise, if it were required to find a third Proportional to the same Number 2 and 4, which may bear the same proportion to 2, that 2 bears to 4; extend the Compasses upon the second Part of the Mean Line of Numbers from 4 to 2 downwards; this done, if you apply that extent from 2 the same way (*viz.* downwards) the movable Point will fall upon 1, the third Proportiona required; And from 1 upon $\frac{5}{10}$ or .5, by the last Corallary of the third Chapter, and from .5 to .25, by the same Corallary, *&c.*

In like manner, if the two Numbers given were 10 and 9, the Compasses being extended downwards from 10 at the upper end of the same Line of Numbers to 9, and that extent applied from 9 the same way, the movable Point of the Compasses will rest upon 8.1, the third Proportional (for the given Numbers being 10 and 9, common sense tells me that it cannot be 81, and therefore ought to be 8.1) and from 8.1 the movable Point will fall upon

on 7.29, the fourth Proportional, *&c.* So likewiſe if the Numbers propounded were 1 and 9, conceiving 10 at the upper end of the Line to repreſent 1, extend the Compaſſes from thence to 9, which extent being applied downwards from 9, will cauſe the movable Point of the Compaſſes to fall upon 81, the third Proportional, and from 81 upon 729, the fourth Proportional, *&c.* And therefore *note* hence, that 1 at the beginning, 1 in the middle, and 10 at the end of the Line, are all arbitrary Points, and may each of them repreſent ſometimes 1, ſometimes 10, ſometimes 100, ſometimes 1000, *&c.* as the terms by which you are to work, ſhall require, according to the third Corollary of the third Chapter.

Neverthelefs neither do the *Examples* before produced, nor thoſe, which ſhall follow in the enſuing Problems at all croſs that which hath been formerly taught in the ſecond Corollary of the third Chapter: For, in the laſt *Example*, the end of the Line in regard of the firſt term given (*viz.* 1) hath the ſingle value of an Unite; but in reſpect of the ſecond term 9 it challengeth the value of 10; and in reference to the third Number 81, the value of 100, *&c.*

Laſtly, if the Numbers given were 10 and 12, the third Proportional upwards would be 14.4, the fourth 17.28, *&c.* and the Number 1 and 12 being propounded, the third Proportional upwards (as before) will be 1443, the fourth 1728, *&c.*

The like Operations may be alſo performed (and that much more exactly) upon the great Line of Numbers: For *Example*, 1 and 4 being given, I deſire to know a third, a fourth, a fifth, *&c.* Geometrically Proportional: To perform this, extend the Compaſſes upon that Line acroſs from 1 at the beginning of the ſecond Part thereof unto 4 upon the firſt part of the ſame; which done, that extent being

ing applied the ſame way, (*viz.* upwards and acroſs) will reach from 4 upon the firſt Part, unto 16 upon the ſecond, and from thence to 64 upon the firſt Part again, *&c.*

PROBL. 2.

One Number being given to be multiplied by another Number given, to find the Product.

Extend the Compaſſes upon the Line of Numbers from 1 unto the Multiplicator; This done, if you apply that extent the ſame way from the Multiplicand, the moveable Point of the Compaſſes will fall upon the Product required.

1. *Example*, Let the Multiplicator given be 25, and the Multiplicand 30: Here if you extend the Compaſſes upon any of the Lines of Number from 1 unto 25, and then apply that extent the ſame way from 30, the moveable Point of the Compaſſes will fall upon 750, the Product required. So 1. 728, and 25. 6 being propounded to be multiplied, the Product will be found 44. 2.

2. *Example*, The two Numbers given being 45 and 25, I extend the Compaſſes upon the ſecond Part of the Mean Line of Numbers from 1 to 25; Then (becauſe, if I apply that extent the ſame way from 45 upon the ſame Part of that Line, the moveable Point will fall beyond the Line) I apply the ſame extent the ſame way from 45 in the firſt Part thereof; which done, the movable Point will fall upon 1125, the Product deſired: So the two Numbers given, being 1.728, and 64.5, the Product required will be 111. 4.

3. *Example*,

3. *Example,* If 75 and 35 were given to be multiplied, the Compaſſes ought to be extended downwards from 1 to 75, in the firſt Part of the Mean Line of Numbers, or (which is all one) from 10 at the upper end of that Line to 75; for, that extent being applied the ſame way from 35, will cauſe the movable Point of the Compaſſes to fall upon 2625, the Product required.

4. *Example,* If it were required to find the Content of a piece of Ground 8.75 Perches long, and 6.45 broad; becauſe this queſtion is reſolved by multiplying the length by the breadth, I extend the Compaſſes from 10. at the top of the Line to 8.75; then applying that extent the ſame way from 6.45, the movable Point will fall upon 56.4, the Content required, *viz*. 56 Perches and $\frac{4}{10}$ or .4 of a Perch.

And here you may obſerve, that theſe laſt *Examples*, and thoſe that are like unto them, may likewiſe be performed in working upwards; But in ſuch caſes to ſhun too great an extent of the Compaſſes, it is better to begin the Operation from 10 at the top of the Line, and ſo to deſcend downwards according to the Inſtructions before delivered: For, (take this for a General Rule, once for all, that) *All Operations, which are wrought upon the Rule of Proportion, are beſt performed, when the legs of the Compaſſes have the leaſt extenſion.*

Again, becauſe this Problem of *Multiplication*, as alſo (for the moſt part) all the reſt that follow, are reſolved by the finding out of a fourth Number *Geometrically* proportional to three other Numbers given, we will therefore here inſert this other Advertiſement: Whenſoever queſtion is made of finding a fourth proportional to three ſuch Numbers given, for the better conveniency of working upon the Rule, the order of the ſecond and third terms may be changed, ſo that always care be taken, that the

firſt

first Number may still retain the first place: F *Example*, you may say, as 1 is to 25, so is 30 to 750 or as 1 is to 30, so is 25 to 750. And this Rule diligently to be observed in Multiplication, Div sion, the Rule of three direct, the resolution of th Plane and Spherical Triangles, and generally in a Questions of such like Proportions; to the end th in working upon the *Rule of Proportion* we may a ways avoid too great an exension of the Compass and by that means perform the Work the more e actly.

Lastly, here observe, that Multiplication, and a other Questions hereafter produced, which may b wrought upon the Mean Line of Numbers, m likewise be performed upon the Great Line of Nu bers (and that much more exactly) by working e ther outright or across, as the Questions propound shall require; which (I well hope) I may hereaf leave to the discretion of the ingenious Reader t discover, without any further instruction, they bei (indeed) but one and the same *Instrument* represent in differing postures.

PROBL. 3.

A Number being propounded to be di vided by another Number, to fin the Quotient.

Extend the Compasses upon the line of Numbers fr the Divisor to 1; This done, if you apply that e tent the same way from the Dividend, the mov Point will fall upon the Number of the Quotient.

1. *Example*, Let 750 be the Number given to divid

divided by 25, the Divisor: I extend the Compasses downwards from 25 to 1; then applying that extent the same way from 750, at last the movable Point will fall upon 30, the Quotient required.

2. The Number 1125 being given to be divided by 25; I extend the Compasses downwards from 25 to 1, then applying that extent the same way from 1125, the movable Point will fall upon 45, the Quotient required. The same Quotient will also be found, if changing the terms you first extend the Compasses from 25 to 1125, and then apply that extent from 1; for so also shall the movable Point fall upon 45, as before; according to the observation made in the last Problem: In like manner 111.4 being propounded, to be divided by 1.728, the Quotient will be found 64.5.

3. The Number 2625 being propounded to be divided by 75; extend the Compasses upwards from 75; in the first Part of the Mean Line of Numbers to 1, or (which is all one) from 75 in the second Part thereof to 10 at the top of the Line; This done, if you apply that extent the same way from 2625, the movable Point will from thence reach to 35, the Quotient required: So likewise 56.4 being given to be divided by 8.75, the Quotient will be 6.45.

Now to discover of how many Figures any Quotient ought to consist, it will be necessary to observe how many times the Divisor may be written under the Dividend according to the Rules of Division; for, of so many Figures shall the Quotient be composed: for *Example*, 12231 being given to be divided by 27; because the Divisor 27 may (according to the Rules of Division) be written three times under the Dividend 12231 (as may appear by this *Example*) I say, that the Quotient, which is produced by the Division of 12231 by 27 consists of three Figures

12231
27..

For

For, having extended the Compasses downward the second Part of the Mean Line of Numbers fr 27 (the Divisor) to 12231 (the Dividend) and plied that extent the same way from 1, the mo able Point will fall in the first Part upon 453, Quotient of 12231 divided by 27.

PROBL. 4.

To three Numbers given to find fourth in a direct Proportion.

EXtend the Compasses from the first Number or T given, unto the second; which done, that extent ing applied the same way from the third Ter will cause the movable Point to fall upon the fo Term required.*

Example, If the circumference of a Circle, wh Diameter is 7, be 22; what circumference wil Circle have, whose Diameter is 14? Extend t Compasses upwards upon the Mean Line of Nu bers from seven in the first Part thereof, unto 14 the second; This done, that extent being applied same way from 22, will make the movable Point upon 44, the circumference required.

Or otherwise downwards; The circumference a Circle being 22, and the Diameter thereof 7, h much shall the Diameter of a Circle be, whose c cumference is 44? Extend the Compasses dow wards from 22 in the second Part, to 7 in the fi which done, that extent being applied the sa way from 44, will reach to 14, the Diameter soug for.

PROBL

PROBL. 5.

To three Numbers given, to find a fourth in an inversed Proportion.

EXtend the Compasses upon the Line of Numbers from the first of the Numbers given to the second, having both the same Denomination; this done, if that extent be applied quite backwards from the third given Number, the movable Point will fall upon the fourth Number you look for.

Example, If 60 Pioners can make a Trench of a certain length and breadth in 45 hours, how long will it be before 40 men can make such another? Extend the Compasses from 60 to 40 (those Terms having both the same Denomination, *viz.* of men.) This done, that extent being applied backwards from 45, will reach to 67. 5, the fourth Number you look for; I conclude therefore that 40 men will perform as much in 67 hours and an half, as 60 men will do in 45 hours.

PROBL. 6.

To three Numbers given, to find a fourth in a double Proportion.

THe use of this Problem appears chiefly in Proportions of Lines to Superficies, or of Superficies to Lines.

Now

Now if the Denomination of the first and second terms be of Lines, *Extend the Compasses upon the Line of Numbers, from the first term to the second; this done, that extent being applied twice the same way from the third term will cause the movable Point to fall upon the fourth term required.*

Example, If the Content of a Circle whose Diameter is 14 inches, be 154, what will the Content of a Circle be, whose Diameter is 28? Here 14 and 28 having the same Denomination (*viz.* of Lines) I extend the Compasses from 14 to 28; then applying that extent the same way from 154, the movable Point will first fall upon 308, and from hence upon 616, the Content desired.

But if the first two terms have the Denomination of *areas* or Contents, and the *quæsitum* be a Line, this is the Rule: *Extend the Compasses upon the Mean Line of Numbers from the first term to the second; this done, that extent being applied the same way upon the Great Line of Numbers from the third term, will cause the movable Point to fall upon the fourth term required.*

Example, If the Diameter of a Circle, whose *area* is 154, be 14; what Diameter will a Circle have whose *area* is 616? Extend the Compasses upon the Mean Line of Numbers from 154 to 616; which done, that extent being applied the same way upon the Great Line of Numbers from 14, will reach to 28, the Diameter required.

PROBL. 7.

To three Numbers given, to find a fourth in a tripled Proportion.

THe use of this Problem appears in the proportion of Lines to Solids, *& contra.*

If therefore the first and second Terms have the denomination of Lines, *Extend the Compasses upon the Line of Numbers from the first Term to the second; this done, and that extent applied three times the same way from the third Term; will cause the movable Point at last to fall upon the fourth Term required.*

If an Iron Bullet, whose Diameter is 4 inches, weighing 9 pounds, what is the weight of another Iron Bullet, whose Diameter is 8 inches? Extend the Compasses from 4 to 8! which done, and that extent applied the same way three times from 9, the movable Point will first fall upon 18, then from 18 upon 36, and at last from 36 upon 72, the weight required.

But if the first two Terms be weights or contents of Solids, and a Line is sought for: *Extend the Compasses upon the Little Line of Numbers from the first Term to the second; This done; and that extent applied the same way upon the Great Line of Numbers from the third term will cause the movable Point of the Compasses to fall upon the fourth Term required.*

If the side of a Cube weighing 72 pounds be 8 inches, how many inches is the side of a Cube that weighs 9 pound? Extend the Compasses downwards upon the Little Line of Numbers from 72 to 9; that done, and the same extent applied the same way upon the Great Line of Numbers from 8, will cause the movable Point to fall upon 4, the side required.

PROBL. 8.

Betwixt two Numbers given to find a Mean Arithmetically Proportional.

THis Problem may be performed without the help of the *Rule of Proportion:* Nevertheless,

 because

because it conduceth to the resolution of the next ensuing Problem, I insert it in this place, and give Rule for it:

Add half the difference of the given Terms to the lesser of them: for, that aggregate is the Arithmetical Mean required.

Example, Let 10 and 40 be the Terms given here, if you substract the one out of the other, the difference will be found 30. whose half (15) being added to 10, the lesser Term, their sum (25) is the Arithmetical Mean you look for.

PROBL. 9.

Betwixt two Numbers given, to find a Mean Musically proportional.

BOethius (*Lib. 2. Arith. cap.* 38.) hath this Rule for it: *Differentiam terminorum in minorem terminum multiplica, & post junge terminos, & juxta eum qui inde confectus est, committe illum numerum, qui ex differentiis & termino minore productus est, cujus cum latitudinem inveneris, addas eam minori termino, & quod inde colligitur medium terminum pones.* Multiply the difference of the Terms by the lesser Term, and likewise the same Terms together: this done, if you divide that Product by the Sum of the Terms, and to the Quotient thereof add the lesser Term the last Sum is the Musical Mean desired.

Or shorter thus:

Divide the Product of the given Term by their Sum, for, this done, the Quotient doubled is the mean required. So the Numbers given being 6 and 12, I say 12 multiply

tiplyed by 6 make 72, which divided by 18 the Sum of 12 and 6) leaves 4 in the Quotient, whose double (8) is the Musical Mean you look for. This Problem therefore may be performed by the second and third aforegoing! or yet otherwise thus:

Find the Arithmetical Mean betwixt the Number given and then the Analogy will be this.

As the Arithmetical Mean found is to the greater Extreme: so is the lesser Extreme to the Musical Mean required.

Example, 10 and 40 being propounded, the Arithmetical Mean betwixt them (by the last Problem) is 25: I say then, As 25 is to 40, so is 10 to 16, the Musical Mean desired: the Term therefore here sought for may be discovered by the fourth Problem aforegoing.

And here (I conceive) it will not be amiss to observe, that by this last Rule, having any two Numbers propounded, you may interject two other Numbers betwixt them! in such sort that they four being in several relations compared one with another, may contain in them all the three Proportions abovementioned, which kind of Harmony *Boetius (lib. 2. cap. ult.)* calls *Maxima & perfecta symphonia:* So in the Numbers before mentioned 10, 16, 25, and 40; if you compare 10, 25, and 40 together, there shall you find *Arithmetical* Proportion; if 10, 16, and 40 together, there Harmony, or *Musical* Proportion; if all of them together, there have you *Geometrical* Proportion discontinued: For as 10 to 16, so 25 to 40. And this is that *Harmony* which the same *Boetius* (in the same place) affirmeth to have *Magnam vim in Musici modulaminis temperamentis, & in speculatione naturalium quæstionum:* Great force in the composure of Musick, and in the discovery of the secrets of Nature: And therefore he also averreth in another place (*viz. lib.* 1. *cap.* 2.) that *the reason of Numbers was the chiefest Rule accor-*

ding to which Almighty God framed the World : According to that testified of the Wisdom of God (in Wisdom of *Sol.* cap. 11. v. 20.) *Thou hast ordered things in Measure; and Number; and Weight.* The S tists also and Politicians fetch much from these Proportions for the regular direction of a well-governed Commonwealth, as may be easily collected out of their Writings, and is learnedly proved by *din* in the last Chapter of his Commonwealth.

PROBL. 10.

Betwixt two Numbers given, to find a Mean Geometrically Proportional.

E*Xtend the Compasses upon the Mean Line of Numbers from one of the Numbers given to the other this done; and the same extent applied upon the Great Line of Numbers from either of those Numbers towards the other; the movable Point will fall in the middle betwixt them; viz. upon the Point representing the Mean Proportional required.*

Example; 8 and 32 being propounded, the Mean Proportional between them will be found 16: For if I extend the Compasses upon the Mean Line of Numbers, from 8 in the first part thereof to 32 in the second, and afterwards apply that extent upon the great Line of Numbers from 8 towards 32, the movable Point will fall upon 16, the Mean Proportional demanded; for as 8 is to 16. so is 16 to 32: so the Mean betwixt 6.4, and 14.4, is 9.6 *&c.*

PROBL. 11

PROBL. 11.

Betwixt two Numbers given to find two Means Geometrically Proportional.

Extend the Compasses upon the Little Line of Numbers from one of the Numbers given to the other: this done, and that extent applied upon the Great Line of Numbers from either of these Numbers towards the other; will cause the movable Point to fall first on the third part of the distance between them; viz. *upon the Point representing one of the Mean Numbers required; and being applied again the same way; will at last rest upon the other Proportional you look for.*

Example; Let 8 and 27 be the two Numbers between which two Mean Proportionals are desired. First, I extend the Compasses upon the Little Line of Numbers upwards from 8 to 27: then applying that extent twice upon the Great Line of Numbers from 8 towards 27, I find the movable Point to fall first upon 12, and then upon 18, which are the two Means you desire to know: for as 8 is to 12, so is 12 to 18, and 18 to 27.

PROBL. 12.

To find the Square-Root of any Number under 1000000.

The *Extraction* of Roots, which is accounted the hardest Lesson in *Arithmetick*; is performed by

he help of this *Instrument* with greatest ease and dexterity: for, whereas the *Problems* before propo- sed, as also those that follow, cannot well be expe- dited without the joynt use of the *Rule* and *Compas- ses* together, these of the *Extraction* of the *Square* and *Cube* Roots may be resolved only by *Inspection* without any trouble at all, or ayd of Compasses; so that a man either riding or going in haste may im- mediately read upon the *Rule* the Root of any *Square* or *Cube* Number propounded: which com- pendious way of *Extraction* cannot choose but prove to be of admirable use, especially in questions that concern Military Orders, as shall more plainly ap- pear hereafter. Wherefore to extract the *Square Root* proceed thus:

1. *When the Figures of the Number given are even,* viz. *when the Number consists of two, four, or six Figures,* look *the same Number in the first part of the Mean Line of Numbers: which done, just at the same Point shall you likewise find upon the Great Line of Numbers the Square Root you look for.*

Example, 264196 being propounded, the Square Root thereof will be found 514: for I find the Number 264196 represented in the first part of the Mean Line of Numbers at the Point x, and at the same Point upon the second part of the Great Line of Numbers I observe 514, the Square-Root re- quired.

2. *When the Figures of the Number given are odd,* viz. *one, three, or five, search the same Number in the second part of the Mean Line of Numbers: which done, just at the same Point upon the Great Line of Numbers shall you find also the Square-Root demanded.*

Example, 144 being propounded, I demand the Square-Root thereof: that Number I find to be re- presented in the second part of the Mean Line of Numbers at the Point s, and just there also upon the Great Line of Numbers I discover 12, which is the

the Square-Root of the Number propounded. So likewise is 144 the Square-Root of 20736.

PROBL. 13.

To extract the Cube-Root of any Number under 1000000000.

When the Number propounded consists of one, four, or seven Figures, find it in the first part of the Little Line of Numbers: that done, at the same Point upon the first part of the Great Line of Numbers, you shall find the Cube-Root you look for.

Example. Let the Number given be 1728 whereof the Cube-Root is required: I find that Number in the first part of the Little Line of Numbers at the Point *t*, and at the same Point upon the Great Line of Numbers I also discover 12, the Cube-Root desired: In like manner is 12,52 the Cube-Root of 1950, and 144 the Cube-Root of 2985984.

2. *When the Number given consists of two, five, or eight Figures, search it in the second part of the Little Line of Numbers, and that proceeding as before, you shall have your desire.*

Example, If 14348907 were given, the Root thereof would be found 243: for, that Number being found in the second part of the Little Line of Numbers at the Point *u*, just at the same Point upon the Great Line I also find 243, the Cube-Root required.

5. *When the Number propounded consists of three, six, or nine Figures, look for it in the third part of the Little Line of Numbers: for so likewise at the same Point upon the Great Line will appear the Root required.*

So the Number 159220088 being found in the first part of the Little Line of Numbers at the Poi z, his Cube-Root is there likewise found upon the Great Line of Numbers to be 542: And the Cub Root of 159220. is found to be 54. 2, &c.

The order of finding out the Cube-Numbers upon the several parts of the Line may be fitly expressed by this Figure

1	2	3
1	2	3
4	5	6
7	8	9

CAP. V.

The Use of the Rule of Proportion in Geometry, viz.

In the Dimension,

1. Of Plain Triangles.

PROBL. 1.

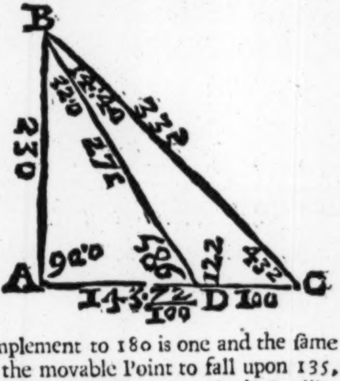
The three Angles and one Side being known, to find the other two Sides.

TO resolve this Problem this is the *Analogy*. As the Sine of the Angle opposed to the side known

known is to the parts of the ſame ſide: ſo is the Angle oppoſed to one of the ſides unknown, to the parts which meaſure that ſide: And therefore

Extend the Compaſſes acroſs from the Sine of the Angle oppoſed to the ſide known; to the ſame ſide; found upon the Mean Line of Numbers: then applying that extent the ſame way from the Sine of the Angle oppoſed to one of the ſides required; the movable Point will fall upon the parts which meaſure that required ſide.

Example In the Triangle *C, B; D;* let the Angle *C* be 43 *degr.* 20 *min.* the l Angle *D* 122 *d.* and by conſequent the Angle *B* (being the Complement of the two other Angles to 180 *d.* or two right Angles) 14 *degr.* 40 *min.* and let the ſide *D; C,* being 100 paces repreſent the diſtance between the two ſtations *D* and *C:* I demand then the diſtance between *C* and *B:* Extend the Compaſſes acroſs from 14 *degr.* 40 *m.* upon the Line of Sines to the middle of the Mean Line of Numbers repreſenting 100, then that extent being applied the ſame way from 122 *d.* upon the Line of Sines or (which is all one) from 58 *degr.* (for by the Rules of *Trigonometry* the Sine of an obtuſe Angle and that of his Complement to 180 is one and the ſame Line) will cauſe the movable Point to fall upon 135, and ſo many paces is the diſtance required: In like manner, the extent being applied the ſame way from 43 *d.* 20 *m.* upon the Line of Sines, the movable



Point will fall upon 271, the parts of the side *D, B.*

Or otherwise, by changing the Terms of the *Analogie*, thus:

Extend the Compasses outright upon the Line of Sines from 14 *d.* 40 *m.* to 58 *d.* then applying that extent the same way upon the Line of Numbers from 100, the moveable Point will rest upon 335, the distance required: so likewise the Compasses being extended outright upon the Line of Sines from 14 *d.* 40 *m.* to 43 *d.* 20 *m.* and that extent applied the same way upon the Line of Numbers from 100, the moveable Point will fall upon 271, the parts of the side *D, B.*

And here observe, that not only this present Problem, but also all those that follow (which concern the resolution of Triangles) may be resolved two manner of wayes, *viz.* by working either outright or across, except some few, which we intend to mark in their proper places. Remember likewise what hath been before touched in the second Chapter aforegoing, *viz.* that the Mean Line of Numbers is the only Line to be used with these of Sines and Tangents, and no other.

PROBL. 2.

By the Knowledge of two Sides and an Angle opposed to one of them, to find the other two Angles and the third Side.

THis is the *Inverse* of the last Problem: for, as the side opposed to the given Angle is, to the Sine of the same Angle; so is the other side known,

known, to the Sine of the Angle thereunto opposed: And therefore

Extend the Compasses across from the parts of the side opposed to the Angle known, unto the Sine of the same Angle: then that extent being applyed the same way from the parts of the other known side, will cause the movable Point to fall upon the Sine of the Angle required.

So in the foresaid Triangle *C, B, D*, the side *C, B*, being 335, the Angle *D* (opposed thereunto) 122 *d.* 0 *m.* and the side *D, C,* 100, the Angle *B* will be found 14 *d.* 40 *m.* For if you extend the Compasses across from 335 upon the Line of Numbers, to 122 *d.* 0 *m.* (or rather to 58 *d.* 0 *m.* as aforesaid) upon the Line of Sines, and after apply that extent the same way from 100 upon the Line of Numbers, the moveable Point will rest upon 14 *d.* 40 *m.* the measure of the Angle *B* required.

Now having the knowledge of two Angles, the other may be easily discovered, being the Complement of those two to 180, as aforesaid: And the Angles being known, the other side may be also found by the Problem aforegoing.

PROBL. 3.

By the Knowledge of two Sides and the Angle included, to find the other two Angles and the third Side.

IF the Angle included be a right Angle, this is the *Proportion:* as the greater side is to the lesser; so is the Tangent of 45 *d.* 0 *m.* to the Tangent of the lesser Angle. And therefore

Extend

Extend the Compasses upon the Line of Numbers downwards from the greater to the less side: then if you apply that extent upon the Line of Tangents the same way from 45 d. *the movable Print will fall upon the Tangent of the lesser Angle.*

Example; In the Rectangle Triangle, *A*; *B*; *D*; of the Diagram aforegoing, the side *A*; *B*; being 230, and the side *A*. *D*, 143. 72; the Angle *B* will be found 32 *d*. 0 *m*. For, if you extend the Compasses downwards upon the Line of Numbers from 230 to 143. 72, that extent being applied the same way from 45 *d*. at the top of the Line of Tangents, will cause the movable Point to fall upon 32 *d*. 0 *m*. *viz*. the measure of the Angle *B*; whose Complement 58 *d*. 0 *m*. is the measure of the Angle *D*: And now the three Angles being thus discovered, the third side may also be known by the first Problem of this Chapter.

But if the included Angle be Oblique, *viz*. either obtuse or acute, then this is the *Analogy*: As the Sum of the sides known is, to the difference of the same sides: so is the Tangent of the half Sum of the Angles unknown, to the Tangent of half their difference: And therefore

Extend the Compasses upon the Line of Numbers downwards and upright from the Sum of the given sides, unto their difference: then applying that extent upon the Line of Tangents from the half Sum of the Angles unknown; the movable Point will fall upon the Tangent of half their difference; which being added unto the said half Sum; make up the greater, but being deducted from it discovers the lesser of the Angles you look for.

An *Example* of this Problem, when the moity of the Angles opposed exceeds not 45 *d*.

In the Triangle *B*; *C*; *D*; the side D, B, being 271, the side *D*; *C*; 100, and the Angle *D*; 122 *d*. the Angle *B* will be found 14 *d*. 40 *m*. and the Angle *C*; 43 *d*. 20 *m*. For, if you extend the Compasses upon the Mean

Mean Line of Numbers downwards from 371 (the Sum of the sides known) to 171 (their difference) that extent being applied the same way upon the Line of Tangents from 29 *d.* (half the Sum of the Angles *B* and *C*, the movable Point will fall upon 14 *d.* 20 *m.* which being added to 29 *d.* amounts to 43 *d.* 20 *m.* for the Angle *C*; and being substracted out of them, the remainder is 14 *d.* 40 *m.* For the Angle *B.*

Two other *Examples* of this Problem, when the moity of the Angles opposed exceeds 45 *d.*

1. In the same Triangle *C*; *B*; *D*; the side *C*; *B*; being 335, the side *C*; *D*; 100; and the Angle *C*; 43 *d.* 20 *m.* the Angle *D* will be 122 *d.* and the Angle *B* 14 *d.* 40 *m.* For; if you extend the Compasses upon the Line of Numbers downwards from 435 (the Sum of the sides known) to 235 (their difference) that extent being applied upon the Line of Tangents backwards (*viz.* upwards) from 68 *d.* 20 *m.* (the half Sum of the Angles *D* and *B* required) the movable Point will fall upon 53 *d.* 40 *m.* which being added to 68 *d.* 20 *m.* their Sum is 122 *d.* 0 *m.* *viz.* the measure of the Angle *D*; and being deducted out of the same 68 *d.* 20 *m.* the remainder is 14 *d.* 40 *m.* the Angle *B.*

2. The side *B*; *C*, being 335, the side *B*; *D*; 271; and the Angle *B* 14 *d.* 40 *m.* I demand the Angles *D* and *C*: the Sum of the sides *B*; *C*; and *B*; *D*; is 606, their difference is 64, and the Angle *C* being 14 *d.* 40 *m.* the Sum of the Angles opposed and unknown is 165 *d.* 20 *m.* and half that is 82 *d.* 40 *m.* Now to satisfie this demand, I extend the Compasses upon the Line of Numbers downwards from 606 to 64: then, because if I apply that extent upon the Line of Tangents backwards (*viz.* upwards, as before) from 82 *d.* 40 *m.* the movable Point will fall as far beyond the top of that Line, as the Term I look for is situate on this side, I apply that extent down-

downwards from 45 *d.* 0 *m.* causing the movable Point also to fall upon the same Line: that done, and the movable Point remaining there fixed, I close the Compasses till the other Point may rest upon 82 *d.* 40 *m.* And having the Compasses so extended, if applying that extent downwards, I set one of the Points at 45 *d.* the other will reach to 39 *d.* 20 *m.* which being added to 82 *d.* 40 *m.* amounts to 122 *d.* *viz.* the Angle *D*: but being deducted out of 82 *d.* 40 *m.* the remainder is 43 *d.* 20 *m.* *viz.* the measure of the Angle *C*.

And in these three Cases having discovered the three Angles, the other side may be likewise found by the first Problem of this Chapter: Observe also that these two last *Examples* will not admit of *Cross-work*: and therefore are Exceptions to the General Rule delivered in the end of the same Problem.

PROBL. 4.

The three Sides being known, to find the Perpendicular, and the three Angles.

The greatest side being assigned for the Base, upon which the Perpendicular shall be supposed to fall, find the Sum and the difference of the other sides: that done, the *Proportion* will be this: As the Base is to the Sum of the other sides, so is the difference of the other sides to a fourth Number which being deducted out of the Base, the Perpendicular will fall in the middle of that which remains: And therefore

Extend the Compasses upon the Line of Numbers from the parts of the Base unto the Sum of the parts of the other side: this done, and that extent applyed the same way from

from the difference of the other sides, will cause the moveable Point to fall upon a fourth Number, which if you substract out of the intire Base, the Perpendicular will fall in the middle of the remainder.

Example, in the Triangle *E, F, G*, the side *E, F*, being 13, the side F, *G*, 11, and the Base *E, G*, 20, I demand the Point of the Base, where the Perpendicular ought to fall, and then the three Angles of the same Triangle: The Sum of the sides is 24, and their difference is 2: I extend therefore the Compasses upon the Line of Numbers from 20 to 24: that done, in this *Example* (because by the third *Corollary* of the first Problem of the third Chapter, the Numbers 20 and 2 are both represented at the same Point) you may observe (without any farther search) the movable Point to discover the parts of the Segment *E, C, viz.* 2. 4, which being deducted out of 20, there remains 17, 6, whose half is 8, 8, which are the parts of the Base comprehended betwixt *C* and *A*, or betwixt *A* and G: I conclude therefore that *A* is the Point of the Base where the Perpendicular ought to fall. Now in the Triangle *A,F,G*, the sides *A, G*, and *G, F*, being known, as also the Angle *F, A, G*, (which is a right Angle by the 10. *Def.* of the 1. *El.* of *Eucl.*) the Angles *G*, and *F*, as also the Perpendicular *F, A*, may be found by the 1 and 2 Probl. of this Chapter. In like manner in the Triangle *E, F, A*, the sides *E, A*, and *E, F*, as also the Angle *E, A, F*, being known, the Angles *E*, and *F*, may be found by the 2. Probl. of

of this Chapter. And lastly, if you add the Angle *E, F, A,* and *A, F, G,* together, their aggregate will make up the Angle *E; F; G:* And so by the knowledge of the three sides have you all the parts of the Triangle throughly resolved.

PROBL. V.

The three Sides being known, to find the Area, or Superficial Content.

FRom the half Sum of the three sines deduct each side, to the end you may discover the difference betwixt the said half Sum and each side: that done, the *Proportions* will be as followeth:

1. *As* 1 *is to the first difference; so is the second difference to a fourth Number.*

2. *As* 1 *is to that fourth Number, so is the third difference to a sixth Number.*

3. *As* 1 *is to that sixth Number, so is the half Sum to an eighth Number, whose Square-Root is the Area required.*

Example; The three sides of the foresaid Triangle *E, F; G,* being 20, 13, and 11, their Sum is 44, half thereof is 22, and the differences betwixt each side and that half are 2, 9, and 11: The operation being thus prepared (because the Number required is a Square-Root) I extend the Compasses upon the Mean Line of Numbers upwards from 1 to 2: then that extent being applied the same way from 9 (in the first part of that Line) the movable Point will fall upon 18 the fourth Number: this done, and the movable Point remaining there fixed, close the Compasses till the other Point fall again upon 13: for that extent being applied from 11, will cause the

the movable Point to fall upon 198, the sixth Number: again, the movable Point remaining there fixed, as before, open the Compasses till the other Point may yet again fall upon 1, and may intercept between the Legs the distance betwixt 1, and 198: for that done, if you apply the same extent (in the first part of the same Line) from 22, the movable Point will fall upon 4356, whose Square-Root (by the 12. Probl. of the last Chapter) will appear at the same Point upon the Great Line of Numbers to be 66, which is also the *Area* required.

2. Of Spherical Rectangle Triangles.

PROBL. 6.

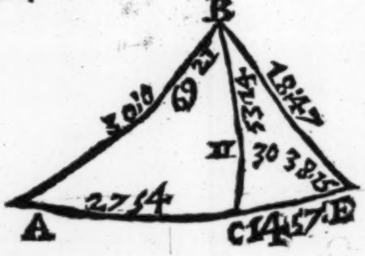
The two Sides being given, to find the Base.

IN Spherical Rectangle Triangles, the side which subtends the right Angle, is called the *Base*, which to find by the knowledge of the other sides, use this *Analogy* following:

As the *Radius* or Sine of 90 *d.* is to the Sine of the Complement (otherwise called the Co-sine) of one of the sides: so is the Co-sine of the other side to the Co-sine of the *Base*: And therefore

Extend

Extend the Compasses downwards uppon the Line of Sines from 90 d. to the Co-Sine of one of the sides: then applying that extent the same way from the Co-Sine of the other side, the movable Point will rest upon the Co-sine of the Base required.



Example. In the Triangle *A, B, C,* the side *A, C* being 27 *d.* 54 *m,* and the side *C, B,* 11 *d,* 30 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.* for if you extend the Compasses downwards from 90 *d.* to 62 *d.* 6 *m.* (the Complement of 27 *d.* 54 *m.* and after apply that extent the same way from 78 *d.* 30 *m.* (the Complement of 11 *d.* 30 *m.*) the movable Point will fall upon 60 *d.* being the Complement of 30 *d.* the *Base* required.

PROBL. 7.

The two Sides being known, to find either of the Oblique Angles.

As the Sine of the side next the Angle required is to the *Radius:* so is the Tangent of the opposite side to the Tangent of the same Angle. And therefore

1. When

1. When the ſide oppoſed to the Angle required exceeds not 45 d. *Extend the Compaſſes upon the Line of Sines from the Sine of the ſide adjacent to the Angle required, to* 90 d. *then that extent being applied the ſame way upon the Line of Tangents, from the Tangent of the ſide oppoſed to the required Angle, the movable Point will fall upon the Tangent of the ſame required Angle.*

1. *Example*, In the ſaid Triangle *A,B,C*, the ſide *A,C*, being 27 *d.* 54 *m.* and the ſide *C,B*, 11 *d.* 30 *m.* I demand the Angle *A.* Extend the Compaſſes upon the Line of Sines from 27 *d.* 54 *m.* to 90 *d.* then that extent being applied the ſame way upon the Line of Tangents from 11 *d.* 30 *m.* the movable Point will reſt upon 23 *d.* 30 *m.* the Angle *A* required.

Or otherwiſe thus: Extend the Compaſſes acroſs from 27 *d.* 54 *m.* upon the Line of Sines to 11 *d.* 30 *m.* upon the Line of Tangents: then applying that extent the ſame way from 90 *d.* upon the Line of Sines, the movable Point will fall upon the Line of Tangents at a Point repreſenting 23 *d.* 30 *m.* as before. And *note*, that in this caſe the Term required will always fall out to be leſs then 45 *d.*

2. *Example*, To know the Angle *B*: Extend the Compaſſes upon the Line of Sines from 11 *d.* 30 *m.* to 90 *d.* then (becauſe that extent being applied upon the Line of Tangents the ſame way from 27 *d.* 54 *m* will cauſe the movable Point to fall as far beyond the top of that Line, as the Term required is ſituate on this ſide) apply the ſame extent backwards upon the Line of Tangents from 45 *d.* cauſing the movable Point to fall alſo upon the ſame Line: for, that done, and the movable Point remaining fixed at the Point where it falls, cloſe the Compaſſes till the other Point may fall upon 27 *d.* 54 *m*, And at laſt that extent being applied outright upon the Line of Tangents from 45 *degr.* will cauſe the moveable Point to reſt upon 69 *d.* 21 *m.* the Angle *B* required. Or otherwiſe: Extend the Compaſſes a-

croſs

cross from 11 *d.* 30 *m.* upon the Line of Sines to 2[illegible] *d.* 54 *m.* upon the Line of Tangents: then if yo[u] apply that extent backwards from 90 *d.* upon th[e] Line of Sines, the movable Point will fall upon th[e] Line of Tangents at a Point representing 69 *d.* 2[illegible] *m.* as before. And here the required Angle is alwa[yes] greater than 45 *d.*

2. When the side opposed to the Angle requi[red] exceeds 45 *d.* *Extend the Compasses upon the Line [of] Sines from the Sine of the side adjacent to the Angle re[quired], to 90 d. That done, if you apply that exte[nt] backwards upon the Line of Tangents from the Tangent [of] the side opposed to the said required Angle, the movab[le] Point will fall upon the Tangent of the same Angle.*

Example, In the *Diagram* annexed, the side *A, C,* being 61 *d.* 53 *m.* and *B, C,* 54 *d.* 28 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, the Compasses being extended upon the Line of Sines from 61 d. 53 *m.* to 90 *d.* and that extent applied backwards upon the Line of Tangents from 54 *d.* 28 *m.* the movable Point will fall upon 57 *d* 47 *m.* the Angle *A* required. And here *observe* 1. that in *Examples* of this kind you cannot work across: 2[.] The Angle here found is alwayes greater tha[n] 45 *d.*

PROBL. 8.

PROBL. 8.

The Base and one of the Oblique Angles being given, to find the other Oblique Angle.

As the *Radius* to the Co-sine of the *Base*; so is the Tangent of the Angle known to the Cotangent of the Angle required: And therefore

1. When the Angle given exceeds not 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Base: then if you apply that extent the same way upon the Line of Tangents from the Tangent of the Angle given; the movable Point will fall upon the Co-tangent of the required Angle.*

Example, In the *Diagram* of the sixth Probl. the Base *A, B,* being 30 *d.* and the Angle *A* 23 *d.* 30 *m.* the Angle *B* will be found 69 *d.* 21 *m.* For, if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* (the Complement of the Base) and that extent applied the same way upon the Line of Tangents from 23 *d.* 30 *m.* the movable Point will rest upon 20 *d.* 39 *m.* whose Complement (found also at the same Point) is 69 *d.* 21. *m.* the Angle *B* required. Or otherwise by cross-work, thus: Extend the Compasses from 90 *d.* upon the Line of Sines to 23 *d.* 30 *m.* upon the Line of Tangents: then that extent being applied the same way from 60 *d.* upon the Line of Sines, the movable Point will fall upon the Line of Tangents at the Point representing 20 *d.* 39 *m.* as before. And here *observe*, that (in this case) the Angle you look for is alwayes less than 45 *d.*

2. When

2. When the Angle given is greater than 45
Extend the Compasses upon the Line of Sines from 90 to the Co-sine of the Base: this done, if you apply the extent upon the Line of Tangents backwards from the Tangent of the Angle given, the movable Point will fall upon the Co-tangent of the Angle required.

1. *Example*, In the *Diagram* of the sixth Probl. *A*, being 30 *d.* and the Angle *B* 69 *d.* 21 *m.* the Angle *A* will be found 23 *d.* 30 *m.* For if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* and that extent applied backwards upon the Line of Tangents from 69 *d.* 21 *m*, the movable Point will fall upon 66 *d.* 30 *m.* the Complement of 23 *d.* 30 *m.* the Angle *A* required. And in this case you cannot use cross-work, and the last Term found upon the Rule is alwayes greater than 45 *d.* but the Term required less.

2. *Example*, In the *Diagram* produced in the last Probl. *B*, *A*. being 74 *d.* 6 *m.* and the Angle *B* 66 *d.* 30 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, if you extend the Compasses upon the Line of Sines from 90 *d.* to 15 *d.* 54 *m.* and then (because that extent being applied backwards, as before, upon the Line of Tangents from 66 *d.* 30 *m.* will cause the movable Point to fall beyond that Line) if you proceed as you were directed in the second *Example* of the said last Probl. at last the movable Point will rest upon 32 *d.* 13 *m.* the Complement of the Angle *A* required. Or otherwise by cross-work: Extend the Compasses from 90 *d.* upon the Line of Sines to 66 *d.* 30 *m.* upon the Line of Tangents: This done, if you apply that extent backwards from 15 *d.* 54 *m.* upon the Line of Sines, the movable Point will rest upon the Line of Tangents at the Point representing 32 *d.* 13 *m.* as before. And (in this case) the last Term found upon the Rule is alwayes less than 45 *d.* but the Term required greater.

PROBL

PROBL. 9.

The Base and one of the Oblique Angles being known, to find the Side adjacent to the same Angle.

AS the *Radius* is to the Co-fine uf the Angle known; ſo is the Tangent of the Baſe to the Tangent of the ſide required: And therefore,

1. When the Baſe is leſs than 45 *d.* *Extend the Compaſſes upon the Line of Sines from* 90 d. *to the Co-ſine of the Angle known: then applying that extent the ſame way upon the Line of Tangents from the Tangent of the Baſe, the movable Point will fall upon the Tangent of the ſide required.*

So in the *Diagram* of the ſixt Problem, *B, A,* being 30 *d.* and *A* 23 *d.* 30 *m.* the ſide *A, C,* (whether you work outright or acroſs) will be found 27 *d.* 54 *m.* And in this caſe the Term required is alwayes leſſer than 45 *d.*

2. When the Baſe exceeds 45 *d.* *Extend the compaſſes upon the Line of Sines from* 90 d. *to the Co-ſine of the Angle known, as before: that done, if you apply the ſame extent upon the Line of Tangents backwards from the Tangent of the Baſe, the movable Point will reſt upon the Tangent of the ſide required.*

So in the *Diagram* produced in the ſeventh Problem, *B, A,* being 74 *d.* 6 *m.* and the Angle *A* 57 *d.* 47 *m.* the ſide *A. C.* will be found 61 *d.* 53 *m.* And in this caſe you cannot work acroſs, and the ſide to be found will be always greater than 45 *d.*

Now if in applying the extent of the Compaſſes from the Tangent of the Baſe, the movable Point

falls

2. When the Angle given is greater than 45 d. *Extend the Compasses upon the Line of Sines from 90 d. to the Co-sine of the Base: this done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the Angle given, the movable Point will fall upon the Co-tangent of the Angle required.*

1. *Example*, In the *Diagram* of the sixth Probl. *B*, *A*, being 30 *d.* and the Angle *B* 69 *d.* 21 *m.* the Angle *A* will be found 23 *d.* 30 *m.* For if the Compasses be extended upon the Line of Sines from 90 *d.* to 60 *d.* and that extent applied backwards upon the Line of Tangents from 69 *d.* 21 *m*, the movable Point will fall upon 66 *d.* 30 *m.* the Complement of 23 *d.* 30 *m.* the Angle *A* required. And in this case you cannot use cross-work, and the last Term found upon the Rule is alwayes greater than 45 *d.* but the Term required less.

2. *Example*, In the *Diagram* produced in the last Probl. *B*, *A*. being 74 *d.* 6 *m.* and the Angle *B* 66 *d.* 30 *m.* the Angle *A* will be found 57 *d.* 47 *m.* For, if you extend the Compasses upon the Line of Sines from 90 *d.* to 15 *d.* 54 *m.* and then (because that extent being applied backwards, as before, upon the Line of Tangents from 66 *d.* 30 *m.* will cause the movable Point to fall beyond that Line) if you proceed as you were directed in the second *Example* of the said last Probl. at last the movable Point will rest upon 32 *d.* 13 *m.* the Complement of the Angle *A* required. Or otherwise by cross-work: Extend the Compasses from 90 *d.* upon the Line of Sines to 66 *d.* 30 *m.* upon the Line of Tangents: This done, if you apply that extent backwards from 15 *d.* 54 *m.* upon the Line of Sines, the movable Point will rest upon the Line of Tangents at the Point representing 32 *d.* 13 *m.* as before. And (in this case) the last Term found upon the Rule is alwayes less than 45 *d.* but the Term required greater.

PROBL

PROBL. 9.

The Base and one of the Oblique Angles being known, to find the Side adjacent to the same Angle.

AS the *Radius* is to the Co-sine of the Angle known; so is the Tangent of the Base to the Tangent of the side required: And therefore,

1. When the Base is less than 45 *d.* *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known: then applying that extent the same way upon the Line of Tangents from the Tangent of the Base, the movable Point will fall upon the Tangent of the side required.*

So in the *Diagram* of the sixt Problem, *B, A,* being 30 *d.* and *A* 23 *d.* 30 *m.* the side *A, C,* (whether you work outright or across) will be found 27 *d.* 54 *m.* And in this case the Term required is alwayes lesser than 45 *d.*

2. When the Base exceeds 45 *d.* *Extend the compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known, as before: that done, if you apply the same extent upon the Line of Tangents backwards from the Tangent of the Base, the movable Point will rest upon the Tangent of the side required.*

So in the *Diagram* produced in the seventh Problem, *B, A,* being 74 *d.* 6 *m.* and the Angle *A* 57 *d.* 47 *m.* the side *A. C.* will be found 61 *d.* 53 *m.* And in this case you cannot work across, and the side to be found will be always greater than 45 *d.*

Now if in applying the extent of the Compasses from the Tangent of the Base, the movable Point falls

falls beyond the Line, work as you were before directed in the second *Example* of the seventh Problem aforegoing, and so shall you also in that case discover the side you look for, which will then always happen to be less than 45 *d.*

PROBL. 10.

The Base and one of the Oblique Angles being known, to find the Side opposed to the same Angle.

As the *Radius* is to the Sine of the *Base*, so is the Sine of the Angle known to the Sine of the Side required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. to the Sine of the Base: For, that extent being applied the same way from the Sine of the given Angle will cause the movable Point to fall upon the Sine of the side required.

Example, In the *Diagram* of the sixth Problem to know the side *B, C,* extend the Compasses upon the Line of Sines from 90 *d.* to 30 *d.* then if you apply that extent the same way from 23 *d.* 30 *m.* the movable Point will fall upon 11 *d.* 30 *m.* the side required.

PROBL. 11

PROBL. 11.

One of the Sides and the Oblique Angle next unto it being known, to find the Base.

As the Co-sine of the Angle known is to the *Radius*; so is the Tangent of the side given to the Tangent of the Base: And therefore,

1. When the side given exceeds not 45 *d. Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *This done, and that extent applied the same way upon the Line of Tangents from the Tangent of the side given, will cause the movable Point to fall upon the Tangent of the Base.* So in the *Diagram* of the sixth Probl. the Angle *A* being 23 *d.* 30 *m.* and the side *A, C,* 27 *d.* 54 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.* But here, if the moveable Point chance to fall beyond the Line, proceed as you have been before directed in the second *Example* of the 7. Probl. And in that case the Term required will alwayes prove greater than 45 *d.*

2. When the given side exceeds 45 *d. Extend the Compasses upon the Line of Sines from the Co-sine of the Angle given, unto* 90 d. *then, if you apply that extent upon the Line of Tangents backwards from the Tangent of the side given, the movable Point will fall upon the Tangent of the Base.* So in the *Diagram* of the seventh Probl. the Angle *A* being 57 *d.* 47 *m.* and the side *A, C,* 60 *d.* 53 *m.* the Base *B, A,* will be found 74 *d.* 6 *m.* And here the Term sought for is always greater than 45 *d.*

PROBL. 12.

One of the Sides and the Oblique Angle next unto it, being known, to find the other Side.

AS the *Radius* is to the Sine of the side given, so is the Tangent of the Angle known to the Tangent of the side required: And therefore

1. When the Angle given exceeds not 45 *d.* Extend the Compasses upon the Line of Sines from 90 unto the Sine of the given side: this done, and that extent applied the same way upon the Line of Tangents from the Tangent of the Angle known; will cause the movable Point to fall upon the Tangent of the side required.* So in the *Diagram* of the sixth Probl. *A, C*; being [...] *d.* 54 *m.* and the Angle *A*, 23 *d.* 30 *m.* the side [...] will be found 11 *d.* 30 *m.* And in *Examples* of this kind cross-work may be used, and the Term sought for is always less than 45 *d.*

2. When the Angle given exceeds 45 *d.* *Extend the Compasses as before: which done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the given Angle, the movable Point will fall on the Tangent of the side required.* So in the *Diagram* of the seventh Probl. *B*; *C*, being 54 *d.* 28 *m.* and the Angle *B*, 66 *d.* 30 *m.* the side *A*, *C.* will be found [...] *d.* 53 *m.* This *Example* and the like cannot be performed by cross-work; and here the Term sought is alwayes greater than 45 *d.* But if in applying the Compasses backwards the movable Point chance to fall beyond the Line, work as you were before directed in the second *Example* of the seventh Problem.

blem of this Chapter, and then will the Term required be alwayes less than 45 *d.*

PROBL. 13.

One of the Sides and the Oblique Angle next unto it being known, to find the other Oblique Angle.

As the *Radius* to the Co-sine of the given Side, so is the Sine of the Angle known, to the Co-sine of the Angle required: And therefore

Extend the Compasses upon the Line of Sines from 90 d. *to the Co-sine of the side given: this done, that extent being applied the same way from the Sine of the given Angle; will reach to the Co-sine of the Angle required.* So in the *Diagram* of the sixth Problem *A*; *C*; being 27 *d.* 54 *m.* and the Angle *A* 23 *d.* 30 *m.* the Angle *B* will be found 69 *d*, 21 *m.*

PROBL. 14.

One of the Sides and the Angle opposed unto it being known, to find the Base.

As the Sine of the Angle given is to the Sine of the side given: so is the *Radius* to the Sine of the Base: And therefore

Extend the Compasses from the Sine of the Angle given

to the Sine of the given side: then if you apply the extent from 90 d. the movable Point will fall upon the Sine of the Base. So in the *Diagram* of the sixth Problem, *A*, being 23 *d.* 30 *m.* and the side *B, C,* 11 *d.* 30 *m.* the Base *B, A,* will be found 30 *d.* 0 *m.*

PROBL. 15.

One of the Sides and the Angle opposed unto it being known, to find the other Oblique Angle.

AS the Co-sine of the side given is to the Co-sine of the Angle given; so is the *Radius* to the Sine of the Angle required: And therefore, *Extend the Compasses from the Co-sine of the given side to the Co-sine of the given Angle: this done, that extent being applied the same way from the* Radius, *will cause the movable Point to fall upon the Sine of the Angle required.* So in the *Diagram* of the sixth Problem the side *A, C,* being 27 *d.* 54 *m.* and the Angle *B,* 69 *d.* 21 *m.* the Angle *A,* will be found 23 *d.* 30 *m.*

PROBL. 16.

One of the Sides and the Angle opposed unto it being known, to find the other Side.

AS the Tangent of the Angle given is to the Tangent of the side given; so is the *Radius* to the

the Sine of the side required: And therefore,

1. When neither the Angle nor side given exceeds 45 *d. Extend the Compasses downwards upon the Line of Tangents from the Tangent of the Angle given, to the Tangent of the side given: this done, the extent being applied the same way upon the Line of Sines from 90 d. will reach to the Sine of the side required.*

So in the *Diagram* of the sixth Problem, the Angle *A* being 23 *d.* 30 *m.* and the side *B, C,* 11 *d.* 30 *m.* the side *A, C,* will be found 27 *d.* 54 *m.*

2. When the Angle and the side given do each of them exceed 45 *d. Extend the Compasses upon the Line of Tangents upwards from the Tangent of the Angle given to the Tangent of the side given, then if you apply that extent backwards upon the Line of Sines from 90* d. *the movable Point will fall upon the Sine of the side required.*

So in the *Diagram* of the seventh Problem, the Angle *B* being 66 *d.* 30 *m.* and the side *A, C,* 61 *d.* 53 *m.* the side *B, C,* will be found 54 *d.* 28 *m.*

3. When the Angle is greater, and the side less then 45 *d. Extend the Compasses upon the Line of Tangents downwards from 45* d. *to the Tangent of the Angle given, then if that extent be applied the same way from the Tangent of the given side, the movable Point will fall upon a Point, which upon the Line of Sines represents the Sine of the side required.*

So in the *Diagram* of the sixth Problem, the Angle *B* being 69 *d.* 21 *m.* and the side *A, C,* 27 *d.* 54 *m.* the side *B, C,* will be found 11 *d.* 30 *m.* And here observe, that *Examples* of this kind may likewise be performed by cross-work, the extent of the Compasses being applied backwards: For, having extended the Compasses across from 69 *d.* 21 *m.* upon the Line of Tangents to 90 *d.* upon the Line of Sines, if you apply that extent backwards and across from 27 *d.* 54 *m.* upon the Line of Tangents, the movable Point will fall upon the Sine of 11 *d.* 30 *m.* the side required.

PROBL. 17.

One of the Sides and the Base being known, to find the Angle opposed to the same Side.

AS the Sine of the Base is to the *Radius*; so is the Sine of the side known to the Sine of the Angle required: And therefore,

If you extend the Compasses from the Sine of the Base unto 90 d. *that extent being applied the same way, will reach from the Sine of the great side unto the Sine of the Angle required.* So in the *Diagram* of the sixth Problem, *B, A*, being 30 *d.* and the side *B, C*; 11 *d.* 30 *m.* the Angle *A* will be found 23 *d.* 30 *m.*

PROBL. 18.

One of the Sides and the Base being known, to find the Oblique Angle adjacent unto that Side.

AS the Tangent of the Base is to the Tangent of the given side; so is the *Radius* to the Co-sine of the Angle required: And therefore,

1. When neither the Base nor the side given exceeds 45 *d. the extent from the Tangent of the Base to the Tangent of the side given, being applied the same way, will reach from* 90 d. *to the Cosine of the Angle required.*

So

So in the *Diagram* of the sixth Problem, the Base ,*A*, being 30 *d.* and the side *A*, *C*, 27 *d.* 54 *m.* he Angle *A* will be found 23 *d.* 30 *m.* And in this case cross-work may also be used, if you apply the Compasses the same way they were extended.

2. When the Base and the side given do each of them exceed 45 *d.* *The extent upwards from the Tangent of the Base to the Tangent of the given side being applied backwards, will reach from* 90 d. *to the Co-sine of the Angle required.*

So in the *Diagram* of the seventh Problem, the Base *B*, *A*, being 74 *d.* 6 *m.* and the side *A*, *C*, 61 *d.* 53 *m.* the Angle *A* will be found 57 *d.* 47 *m.* Howbeit in this case cross-work hath no place.

3. When the Base is greater, and the side less than 45 *d.* *Work as you were taught in the third Rule of the sixteenth Problem aforegoing.*

PROBL. 19.

One of the Sides and the Base being known, to find the other Side.

As the Co-sine of the side given is to the *Radius*; so is the Co-sine of the Base to the Co-sine of the side required: And therefore,

The extent from the Co-sine of the side given to 90 d. *being applied the same way, will reach from the Co-sine of the Base, to the Co-sine of the side required.*

So in the *Diagram* of the sixth Problem the Base *B*, *A*, being 30 *d.* and the side *A*, *C*, 27 *d.* 54 *m.* the side *B*, *C*, will be found 11 *d.* 30 *m.*

PROBL. 20:

The two Oblique Angles being known to find the Base.

AS the Tangent of one of the Angles is to the Cotangent of the other Angle; so is the *Radius* to the Co-sine of the Base: And therefore,

1. When one of the Angles given, and the Complement of the other are each of them less than 45 d. *The extent from the Tangent of the Angle less than 45 d. unto the Co-tangent of the other, will reach from 90 d. to the Co-sine of the Base.* So in the *Diagram* of the sixth Problem the Angle *A* being 23 d. 30 m. and the Angle *B* 69 *d.* 21 *m.* the Base *B*, *A*, will be found 30 *d.* And here cross-work may likewise be used.

2. When one of the Angles is greater, and the Complement of the other less than 45 *d.* *Proceed as you have been taught in the third Rule of the* 16. *Problem aforegoing.*

PROBL. 21.

The two Oblique Angles being known to find either of the Sides.

AS the Sine of one of the Angles is to the Cosine of the other Angle: so is the *Radius* to the Cosine of the side opposite to the Angle whose Co-sine was taken: And therefore,

The extent from the Sine of one of the Angles given, to the Co-sine of the other, being applied the same way, will reach from 90 d. to the Co-sine of the side opposed to the Angle, whose Co-sine was taken.

So in the *Diagram* of the sixth Problem, the Angle *A* being 23 *d.* 30 *m.* and the Angle *B* 69 *d.* 21 *m* the side *A, C,* will be found 27 *d.* 54 *m.*

3. Of Spherical Oblique Angle Triangles.

PROBL. 22.

Two Angles and a Side opposed to one of them being known, to find the Side opposed to the other.

As the Sine of the Angle subtended by the side known is to the Sine of the same side; so is the Sine of the Angle subtended by the side required, to the Sine of that side: And therefore,

The extent from the Sine of the Angle opposed to the side known, unto the Sine of the same side, being applied the same way from the Sine of the Angle opposed to the side required, will reach to the Sine of the side so required.

So in the *Diagram* of the sixth Problem, the Angle *E,* being 38 *d.* 15 *m.* the side *B, A,* 30 *d.* and the Angle *A* 23 *d.* 30 *m.* the side *B, E,* will be found 18 *d.* 47 *m.*

PROBL. 23.

Two Sides and the Angle opposed to one of them being known, to find the Angle opposed to the other Side.

As the Sine of the side subtending the Angle known is to the Sine of the same Angle; so is the Sine of the side subtending the Angle required, to the Sine of that Angle: And therefore,

The extent from the Sine of the side subtending the Angle known, to the Sine of the same Angle, being applied the same way, will reach from the Sine of the side subtending the Angle required, to the Sine of that Angle.

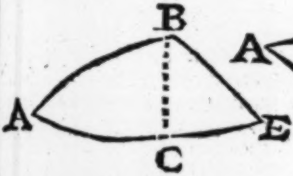
So in the *Diagram* of the sixth Problem, *B, A*, being 30 *d.* the Angle *E* 38 *d.* 15 *m.* and the side *B. E*, 18 *d.* 47 *m.* the Angle *A* will be found 23 *d.* 30 *m.*

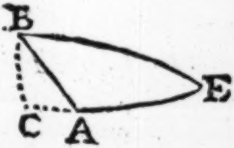
The studious Reader hath by this time (I presume) so well acquainted himself with the turnings and windings of this Instrument, *that in the resolution of most of the ensuing Problems, it will (I conceive) be only necessary to produce the bare* Analogy, *without annexing either Rule or Example as heretofore, and to refer the proper application thereof, to his farther industry and discretion.*

PROBL.

PROBL. 24.

In any of the Triangles annexed, the Sides A, B, *and* A, E, *together with the Angle* A, *being known, to find the Side* B, E,





IN an Oblique Angle Triangle, when the Terms propounded are two ſides and one Angle, or two Angles and one ſide, and yet the Term required undiſcoverable by the two laſt premiſed Problems, you are to convert ſuch a Triangle into two Rectangle Triangles, by ſuppoſing a Perpendicular to be let fall from any one of the Angles upon his oppoſite ſide, in ſuch ſort that two of the Terms propounded may in one of thoſe Rectangle Triangles ſtill remain given and intire; for by this means all the other parts of ſuch a Triangle thus converted, may be readily diſcovered by the *Analogies* of Rectangle Triangles: And the Perpendicular thus imagined, will fall within the Triangle, when the Angles adjacent to the ſide upon

upon which it falls, are of one and the same ki[nd] that is, both acute, or both obtuse; but otherw[ise] without the Triangle, when those Angles are of d[if]fering kinds, *viz.* the one acute and the other obt[use] as plainly appears by the Triangles annexed, in wh[ich] (having the sides *A, B*, and *A, E.* as also the An[gle] *A* propounded) to find the side *B, E*, use these *A[na]logies* following:

1. As the *Radius* is to the Co-sine of *A*; so is t[he] Tangent of *A, B*, to the Tangent of *A C*,

2. As the Co-sine of *A, C*, to the Co-sine of *C, E* so is the Co-sine of *A, B*, to the Co-sine of *B, E*.

And here observe, that to come to the knowled[ge] of *C, E*, in cases that resemble the first of the *Di[a]grams* annexed, having found *A, C*, you are to dedu[ce] it out of *A, E*; again, in such cases as are like th[e] second *Diagram, A, E*, ought to be deducted out o[f] *A, C*; and lastly in those that resemble the thir[d] *Diagram, A, C*, and *A, E*, are to be added togethe[r].

PROBL. 25.

In the same Triangles, A,B, *and* A, E, *together with the Angle* A, *bein[g] known, to find either of the other Angles, and namely (for Example) the Angle* E.

1. As the *Radius* to the Co-sine of *A*; so is the Tangent of *A, B*, to the Tangent of *A, C*.

2. As the Sine of *C, E*, to the Sine of *A, C*; so is the Tangent of *A*, to the Tangent of *E*.

PROBL.

PROBL. 26.

A, B, *and* B, E, *together with* A, *being known, to find* A, E.

1 AS the *Radius* to the Co-sine of *A*; so is the Tangent of *A, B*; to the Tangent of *A, C*.

2. As the Co-sine of *A, B*, to the Co-sine of *B, E*; so is the Co-sine of *A, C*, to the Co-sine of *C, E*.

PROBL. 27.

A, B, *and* B, E, *together with* A, *being known, to find* B.

1. AS the *Radius* to the Co-sine of *A, B*; so is the Tangent of *A*, to the Co-tangent of *A, B, C*.

2. As the Tangent of *B, E*, to the Tangent of *A, B*; so is the Co-sine of *A, B, C*, to the Co-sine of *C, B, E*.

PROBL. 28.

A, *and* B, *together with* A, B, *being known, to find either of the other Sides, and namely (for Example) the Side* B, E.

1. As

1. AS the *Radius* to the Co-sine of *∠A, B*; so is the Tangent of *∠A*, to the Co-tangent of *∠A, B, C*.

2. As the Co-sine of *C, B, E*; to the Co-sine of *∠A, B, C*; so is the Tangent of *∠A, B*, to the Tangent of *B, E*.

PROBL. 29.

A, *and* B, *together with* A, B, *being known, to find* E.

1. AS the *Radius* to the Co-sine of *∠A, B*; so is the Tangent of *∠A*, to the Co-tangent of *∠A, B, C*.

2. As the Sine of *∠A, B, C*, to the Sine of *C; B; E*; so is the Co-sine of *∠A*, to the Co-sine of *E*.

PROBL. 30.

A, *and* E, *together with* A, B, *being known, to find* A, E.

1. AS the *Radius* to the Co-sine of *∠A*; so is the Tangent of *∠A, B*, to the Tangent of *∠A, C*.

2. As the Tangent of *E*, to the Tangent of *∠A*; so is the Sine of *∠A, C*, to the Sine of *C, E*.

PROBL.

PROBL. 31.

A, *and* E, *together with* A, B, *being known, to find* B.

1. AS the *Radius* to the Co-fine of *A, B:* fo is the Tangent of *A*, to the Co-tangent of *A, B, C.*

2. As the Co-fine of *A*, to the Co-fine of *E:* fo is the Sine of *A, B, C*, to the Sine of *C, B, E.*

PROBL. 32.

Three Sides being known, to find any of the Angles.

ADd the three fides together, then from the half Sum thereof fubftract the fide oppofite to the Angle required: this done, the *Proportions* will be as followeth:

1. *As the* Radius *to the Sine of one of the fides including the Angle required: fo is the Sine of the other fide including the fame Angle to a fourth Sine.*

2. *As that fourth Sine is to the Sine of the half Sum of the fide: fo is the Sine of the difference betwixt that half Sum, and the fide oppofed to the Angle required, to a feventh Sine, betwixt which and 90* d. *(at the end of the Line of Sines) if you with your Compaffes difcover the half diftance, that Point fhall reprefent unto you an Ark, whofe Complement being doubled is the Angle you look for.*

So in the *Diagram* of the 6. Problem the fide *A, B*, being

B, being 30 d. the ſide B, E, 18 d. 47 m. and the ſide A, E, 42 d. 51 m. I demand the Angle B: The Sum of the Sides is 91 d. 38 m. half that Sum is 45 d. 49 m. The ſide A. E, being ſubſtracted out of that half, there remains 2 d. 58 m. And therefore to diſcover the Angle B, proceed thus:

Extend the Compaſſes upon the Line of Sines from 90 d. unto 30 d. then applying that extent the ſame way, and upon the ſame Line from 18 d. 47 m. the movable Point will fall upon 9 d. 16 m. Again, that Point remaining there fixed, extend the Compaſſes ſo far that their other Point may reſt upon 45 d. 49 m. this done, and that extent applied the ſame way from 2 d. 58 m. will cauſe the movable Point at laſt to fall upon 13 d. 20 m. whoſe half diſtance towards 90 d. will happen upon a Point repreſenting 28 d. 42 m. whoſe Complement (viz. 60 d. 18 m.) being doubled, amounts to 122 d. 36 m. the quantity of the Angle B required.

PROBL. 33.

The three Angles being known, to find any of the Sides.

IF in ſtead of the greateſt Angle, you take his Complement to 180 d. the Angles convert themſelves into ſides, and the ſides into Angles, and then (by conſequent) the operation will be the ſame with that of the laſt *Problem.*

4. Of

4. Of divers other Geometrical Figures.

PRobl. 34. *The Diameter of a Circle being known, to find the Circumference.*

The extent upon the Line of Numbers from 1 to the Diameter, will reach from 3.142 to the Circumference.

Probl. 35. *To find the Superficial Contnt.*

The extent from 1 to the Diameter being twice repeated from .7854, will reach to the Content O, otherwise thus: The extent upon the Great Line of Numbers, from 1 to the Diameter, will reach upon the Mean Line of Numbers from .7854 to the Content: Or yet thus; the Extent upon the Great Line of Numbers from 1 to .7854 will reach upon the Mean Line of Numbers from the Diameter to the Content. And in this manner may divers of the insuing Problems be diversified, which (as before) I refer to the discretion of the Practioner.

Probl. 36. *To find the side of the Square, which may be inscribed within the same Circle.*

The extent from 1 to .7071 will reach from the Diameter to the side of the Square required.

Probl. 37. *Having the Circumference to find the Diameter.*

The extent from 1 to .3183 will reach from the Circumference to the Diameter.

Probl. 38. *To find the Superficial Content.*

The extent from 1 to the Circumference being twice repeated from .07958, will reach to the Content. Or, &c.

Probl.

Probl. 39. *To find the side of the Square, which may be inscribed within it.*

The extent from 1 to the Circumference, will reach from 2251 to the side of the Square required.

Probl. 40. *Having the Content of a Circle, to find the Diameter.*

The extent from 1 to 1. 273 will reach from the Content to another number, whose Square Root is the Diameter required.

Probl. 41. *To find the Circumference.*

The extent from 1 to 12. 57 will reach from the Content to another Number, whose Square Root is the Circumference required.

Probl. 42. *To find the side of the Square equal unto it.*

Bxtract the Square Root thereof by the 12. Probl. of the last Chapter, and you have your desire.

Probl. 43. *The breadth of a long Square being given in Inch-measure, and the length in Foot-measure, to find the Content in Feet.*

The extent from 12 to the breadth in Inches, will reach from the length in Feet to the Content in Feet. Or, *vice versa*, the extent from 12 to the length in Feet, will reach from the breadth in Inches to the Content in Feet.

Probl. 44. *The breadth and length of a long Square being given in Foot-measure to find the Content thereof in Yards.*

The extent from 9 to the breadth, will reach from the length to the Content in Yards. Or, *&c.*

Probl. 45. *To find the Content in single Perches.*

The extent from 16. 5 to the breadth, will reach from the length to the Content in single Perches. Or, *&c.*

Probl. 46. *To find the Content in Square Perches; otherwise (in* Architecture*) called* Poles.

The extent from 272. 25 to the breadth, will reach from the length to the Content in Poles. Or, *&c.*

Probl.

Probl. 47. *The breadth and length of a long Square being given in Perches, to find the Content in Acres.*

The extent from 160 to the breadth, will reach from the length to the Content in Acres. Or, &c.

Probl. 48. *The breadth and depth of a Square Rectangle solid, being given in Inch-measure, and the length in Foot-measure to find the Content thereof in Feet.*

The extent from 12 to the breadth or depth in Inches, being twice repeated from the length in Feet, will reach to the Content in Feet. Or, &c.

Probl. 49. *The breadth and depth of a Rectangle solid (not just square) being known) in Inch-measure, and the length in Foot-measure to find the Content in Feet.*

Find (by the tenth *Problem* of the last Chapter) the Mean Proportional betwixt the breadth and the depth; for then, the extent from 12 to that Mean Proportional, being twice repeated from the length in Feet, will reach to the Content in Feet.

Probl. 50. *The breadth and depth of a Rectangle solid (not just square) being known in Foot-measure, to find the Base or Superficies at the end thereof.*

The extent from 1 to the breadth, will reach from the depth to the *Base* required.

Probl. 51. *The Base and length of a Rectangle solid being known in Foot-measure, to find the Content in Feet.*

The extent from 1 to the Base, will reach from the length to the Content.

Probl. 52. *Having the Diameter of a Cylinder, to find the Base.*

The Base of a Cylinder being a perfect Circle this *Problem* may be resolved by the 35 aforegoing.

Probl. 53. *The Base and length of a Cylinder being known, to find the Content.*

The extent from 1 to the Base, will reach from the length to the Content.

Probl.

Probl. 54. *Having the* Axis *of a Sphere, to find* [t]*he Superficial Content.*

The extent from 1 to the *Axis*, being twice re[-]peated from 3.142, will reach to the Superficial Con[-]tent required. Or, &c.

Probl. 55. *To find the solid Content.*

The extent from 1 to the *Axis*, being thrice re[-]peated from .5238, will reach to the solid Content re[-]quired. Or, &c.

CAP. VI.

The Use of the Rule of Proportion in Astronomy.

PROBL. 1.

By the Sun's Shadow, to find his height.

THe extent upon the Mean Line of Numbers from the length of the Rules Shadow to the height thereof (held Perpendicular to the Horizon) will reach upon the Line of Tangents from 45 *d.* to the Sun's height required.

Probl. 2. *The Sun's greatest Declination, together with his distance from the next Equinoctial Point being known, to find his present Declination.*

As the *Radius* to the Sine of the Sun's distance from the next Equinoctial Point: so is the Sine

the Sun's greatest Declination to the Sine of the Declination required.

Probl. 3. *To find the Right Ascention.*

As the *Radius* to the Tangent of his distance. &c. so is the Co-sine of his greatest Declination to the Tangent of his Right Ascension.

Probl. 4. *The Sun's greatest Declination, together with his present Declination, being known, to find his Right Ascension.*

As the Tangent of his greatest Declination to the *Radius*, so is the Tangent of his present Declination to the Sine of his Right Ascention.

Probl. 5. *The Elevation of the Pole, together with the Sun's Declination being known, to find how long the Sun riseth or setteth before or after the hour of six.*

As the Co-tangent of the Elevation is to the *Radius*; so is the Tangent of the Sun's Declination to the Sine of the *Ascentional* Difference between the hour of six, and the Sun's rising or setting.

Probl. 6. *To find the Sun's Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Declination; so is the *Radius* to the Sine of the Amplitude.

Probl. 7. *The Elevation of the Pole, the Sun's greatest Declination, and his distance from the next Equinoctial Point being known to find the Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Sun's distance; so is the Sine of the Sun's greatest Declination to the Amplitude required.

Probl. 8. *When the Sun is in the Equinoctial, by knowing the Elevation of the Pole, to find the Sun's height at any time assigned.*

As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Sun's distance from six a Clock to the Sine of the height required.

Probl. 9. *The Elevation of the Pole, and the Declination of the Sine being known, to find the Sun's height at the hour of six.*

As the *Radius* to the Sine of the Latitude; so the Sine of the Declination to the Sine of the heig required.

Probl. 10. *To find the Sun's height at any time a signed.*

1. As the Radius to the Co-tangent of the El vation, so is the Sine of the Sun's distance fro six, to the Tangent of an Ark, which being su tracted out of the Sun's distance from the Pole, say again.

2. As the Co-sine of the Ark found is to th Co-sine of the residue of the Sun's distance from th Pole; so is the Sine of the Elevation to the Sine the height required.

Probl. 11. *To find the time when the Sun will due East and West.*

As the Tangent of the Elevation to the *Radius* so is the Tangent of the Declination to the Co-si of the hour from the Meridian.

Probl. 12. *To find the Sun's height, when he come to be due East and West.*

As the Sine of the Elevation to the *Radius*; so the Sine of the Declination to the height required.

Probl. 13. *To find the Sun's Azimuth at the ho of six.*

As the Co-sine of the Elevation is to the Co tangent of the Declination; so is the *Radius* to th Tangent of the Azimuth from the North part the Meridian.

Probl. 14. *The Complement of Elevation, the Sun distance from the Pole, and the Complement of the Sun height being known, to find the Azimuth.*

Having added the three given Terms togethe find the difference betwixt their half Sum and th Sun's distance from the Pole: this done, the *Pro portion* will be as followeth:

1. As the *Radius* to the Co-sine of the Elevation so is the Co-sine of the height to a fourth Sine:

2.

2. As that fourth Sine is to the Sine of the half Sum; so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Ark, whose Complement being doubled is the *Azimuth* you look for.

Probl. 15. *To find the hour of the Day.*

Having added the three given Terms together, as before, find the difference betwixt their half Sum and the Complement of the Sun's height: this done, the *Proportions* will be these:

1. As the *Radius* to the Co-sine of the Elevation; so is the Sine of the Sun's distance from the Pole to a fourth Sine.

2. As that fourth Sine is to the Sine of the half Sum: so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Ark, whose Complement being doubled and converted into Time, will produce the hour required.

CAP. VII.

The Use of the Rule of Proportion in Dialling.

PROBL. 1.* *To make a direct Polar Dial.*

HAving assigned a Line drawn in the middle of the Plane for the Meridian, and another Line

Line drawn parallel unto it for ſome other hour,whi
may be deſcribed upon the Plane: I ſay,

1. As the Tangent of that hour is to the *Radi*
ſo is the diſtance of that Hour-line from the Me
dian to the height of the Stile.

2. As the *Radius* is to the height of the Stile;
is the Tangent of any other hour, to the diſta
of the ſame hour from the Subſtile.

Probl. 2. *A Meridian Dial.*

Having drawn a Line repreſenting part of
Axis of the World towards a proper ſide of
Plane, (according to his ſituation either Eaſtward
Weſtward) aſſigned that Line for the hour of
the *Proportion* will fall out to be as in the former *Pr*
blem; for,

1. As the Tangent of any hours diſtance fr
ſix is to the *Radius;* ſo is the diſtance of the h
upon the Plane from the Hour-line of ſix, to
height of the Stile.

2. As the *Radius* is to the height of the Stile;
is the Tangent of any other hours diſtance fr
ſix to the diſtance of the ſame hour from the Su
ſtile.

Probl. 3. *An Horizontal Dial.*

As the *Radius* to the Tangent of the hour give
ſo is the Sine of the Elevation to the Tangent of
Hour-line from the Meridian.

Probl. 4. *A Vertical Dial.*

As the *Radius* to the Tangent of the hour: ſ
the Co-ſine of the Elevation of the Tangent of
Hour-line from the Meridian.

Probl. 5. *A Vertical Inclining Dial.*

Having found out the Elevation of the Pole
bove the Plane, according to its inclination, the P
portion will be this:

As the *Radius* to the Tangent of the Hour:
is the Sine of the Elevation above the Plane,to
Tangent of the Hour-line from the Meridian.

Probl. 6. *A Vertical Declining Dial.*

1. As the *Radius* to the Co-tangent of the Elevation : ſo is the Sine of the Declination to the Tangent of the Subſtile diſtance from the Meridian of the Place.

2. As the *Radius* to the Co-ſine of the Declination : ſo is the Co-ſine of the Elevation to the Sine of the Stile's height above the Subſtile.

3. As the Sine of the Elevation is to the *Radius* : ſo is the Tangent of the Declination to the Tangent of the Inclination of the Meridian of the Plane to the Meridian of the Place.

4. As the *Radius* to the Sine of the Stile's height above the Subſtile : ſo is the Tangent of the Angle at the Pole comprehended between the hour given and the Meridian of the Plane, to the Tangent of the Hour-lines diſtance from the Subſtile.

Probl. 7. *A Meridian Inclining Dial.*

1. As the *Radius* to the Tangent of the Elevation : ſo is the Sine of the Inclination to the Tangent of the Subſtile's diſtance from the Meridian.

2. As the *Radius* is to the Sine of the Elevation : ſo is the Co-ſine of the Inclination to the Sine of the Stile's height above the Subſtile.

3. As the Co-ſine of the Elevation is to the *Radius* : ſo is the Tangent of the Inclination, to the Tangent of the Inclination of Meridians.

4. As the *Radius* is to the Sine of the Stile's height above the Subſtile : ſo is the Tangent of the Angle at the Pole, to the Tangent of the Hour-lines diſtance from the Subſtile.

Probl. 8. *A Polar Declining Dial.*

1. As the *Radius* to the Sine of the Declination : ſo is the Co-ſine of the Elevation to the Co-ſine of the Ark comprehended between the Horizon and the Subſtile.

2. As the *Radius* to the Tangent of the Declination : ſo is the Sine of the Elevation to the Tan-

gent of the Inclination of Meridians, which bei converted into time, sheweth how many hours Substile ought to be placed from the Hour-li of 11.

3. As the *Radius* is to the Tangent of the ho distance from the Substile: so are the parts of height of the Stile, to the distance of the Subst from the Hour-line required, measured by a Scale like parts.

Probl. 9. *A Declining Inclining Dial.*

1. As the *Radius* to the Tangent of Inclinati to the Horizon: so is the Co-sine of Declinati to the Tangent of the Ark of the Meridian of Place intercepted between the Horizon and Plane, which being compared with the Elevation the Pole, the distance of the Pole from the Pla may be thereby readily discovered.

2. As the *Radius* is to the Sine of Declinati from the Vertical: so is the Sine of Inclination the Horizon, to the Co-sine of the Inclination to Meridian.

3. As the *Radius* is to the Co-sine of Inclinati to the Horizon: so is the Cotangent of Decli tion to the Tangent of the Ark of the Plane inte cepted between the Horizon and the Meridian of Place.

4. As the *Radius* is to the Sine of the Inclinati to the Meridian: so is the Tangent of the Pole's stance from the Plane, to the Tangent of the Su stile's distance from the Meridian.

5. As the *Radius* is to the Pole's distance fro the Plane: so is the Sine of the Inclination to Meridian, to the Sine of the Stile's height abo the Substile.

6. As the Cosine of the Pole's distance from Plane is to the *Radius*: so is the Cotangent of Inclination to the Meridian, to the Tangent of Inclination of Meridians.

7.

7. As the *Radius* is to the Stiles height above the Subſtile; ſo is the Tangent of the Angle at the Pole, to the Tangent of the Hour-line's diſtance from the Subſtile.

CAP. VIII.

The Uſe of the Rule of Proportion in Geography.

Probl. 1. *Two Places being propounded, which differ only in Latitude, to find their Diſtance.*

WHen the two places are ſituate under the ſame Meridian, and upon the ſame ſide of the Equinoctial; *Subſtract the leſſer Latitude out of the greater; that done, the remainder is the diſtance required.*

2. When one of the places propounded is ſituate upon this ſide the Equinoctial, and the other upon that, and yet both under the ſame Meridian, as before: *Add the two Latitudes together; this done, their ſum is the diſtance required.*

Probl. 2. *Two places, which differ only in Longitude, being propounded, to know their diſtance.*

1. When the Places are both of them ſituate under the Equinoctial: *Subtract the leſſer Longitude out of the greater: this done, the remainder is the diſtance required.*

2. When the Places are situate under some Para[llel] betwixt the Equinoctial and one of the Poles: *Th[en] as the Radius is to the Cosine of the common Latit[ude] given: so is the Sine of half the difference of Longit[ude] to the Sine of half the distance.*

Probl. 3. *Two places being given, which differ b[oth] in Longitude and Latitude, to find their distance.*

1. When one of the Places is situate under the E[-] quinoctial, and the other towards one of the Pole[s:] Then, *As the Radius is to the Co-sine of the differe[nce] of Longitude: so is the Co-sine of the Latitude gi[ven] to the Cosine of the distance required.*

2. When both Places are without the Equinocti[al] and towards one of the Poles: Then, *As the Radi[us] is to the Co-sine of the difference of Longitude: so is [the] Co-tangent of the lesser Latitude to the Tangent of a[no]ther Ark,* which being substracted out of the Co[m]plement of the lesser Latitude, retain the Ark the[n] of remaining; and say again, *As the Co-sine of [the] Ark found is to the Co-sine of the Ark remaining: [so] is the Sine of the lesser Latitude to the Co-sine of the [di]stance required.*

3. When both Places are without the Equinocti[al] and one of them situate towards the North Pole, [and] the other towards the South: say thus, *As the Ra[di]us is to the Co-sine of the difference of Longitude: [so is] the Co-tangent of one of the Latitudes, to the Tang[ent of] another Ark,* which being substracted out of [the] other Latitude, and 90 *d.* added together: say ag[ain,] *As the Co-sine of the Ark found is to the Co-sine of [the] Ark remaining: so is the Sine of the Latitude [first] taken, to the Co-sine of the distance required.*

CAP. IX.

The Use of the Rule of Proportion in Navigation.

Probl. 1. *The Latitudes of two Places being known, to find the Meridional Difference.*

1. WHen one of the Places is situate under the Equinoctial, and the other without: *The Degrees and Decimal Minutes found upon the Scale of Equal Parts at the Point; where that other Latitude is represented upon the Scale of Latitudes, are the Meridional difference required.*

2. When one of the Places have Southerly, and the other Northerly Latitude: *Extend the Compasses upon the Line of Latitude, from the beginning of that Line to the lesser Latitude: that done, if you apply that extent upon the same Line, and the same way from the greater Latitude, the movable Point will discover upon the Line of equal Parts, the Meridional difference desired.*

3. When both Places have Northerly or Southerly Latitude: *Extend the Compasses upon the Line of Latitudes from one of the Latitudes to the other: this*

done, if you apply that extent from the beginning of Line, the movable Point will shew you upon the Scale Equal Parts the Meridional difference you look for.

Probl. 2. *The Latitudes of two places together their difference of Longitude being known, to find Rumbe directing from the one to the other.*

As the Meridional difference is to the difference of Longitude: so is the *Radius* to the Tangent the Rumbe: And therefore,

The extent upon the Mean Line of Numbers from Meridional difference to the difference of Longitude, reach upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe.*

And *note* here, that in this Problem and the like you may make use of the double Scale, placed upon the last Line of the Rule of Proportion, at the end of the Scale of Inches: *viz.* (if need be) for the more speedy reduction of the Sexagenary Minutes of the Longitude into Decimals, & *contra*: to the end you may by that means the more readily work them upon the Mean Line of Numbers.

Probl. 3. *By both Latitudes and Rumbe to find the distance upon the Rumbe.*

As the Co-sine of the Rumbe to the true difference of Latitudes: so is the *Radius* to the distance required: And therefore,

Extend the Compasses across from the Co-sine of the Rumbe (found upon the Line of Sines) to the true difference of Latitudes (found upon the Mean Line of Numbers) this done, if you apply that extent the same way and across from 90 d. *upon the Line of Sines, the movable Point will shew you upon the Mean Line of Numbers (in Degrees and Decimal Minutes) the distance required.*

Probl. 4. *By both Latitudes and Rumbe, to find the difference of Longitude.*

As the *Radius* to the Tangent of the Rumbe so is the Meridional difference of the Latitudes

the difference of Longitude required: And therefore

The extent upon the Line of Tangents from 45 d. to the Tangent of the Rumbe, will reach upon the Mean Line of Numbers from the Meridional difference of the Latitudes to the difference of Longitude required.

Probl. 5. *By both Latitudes and distance to find the Rumbe.*

As the distance is to the true difference of Latitudes: so is the *Radius* to the Co-sine of the Rumbe: And therefore,

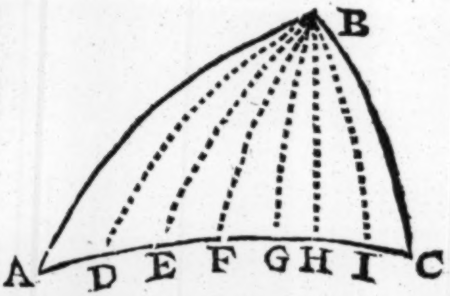
The extent upon the Mean Line of Numbers, from the distance to the difference of Latitudes, will reach upon the Line of Sines from 90 d. to the Co-sine of the Rumbe.

Probl. 6. *By one Latitude, distance, and Rumbe, to find the other Latitude.*

As the *Radius* to the Co-sine of the Rumbe: so is the distance to the true difference of Latitudes: And therefore,

The extent upon the Line of Sines from 90 d. to the Co-sine of the Rumbe, will reach upon the Mean Line of Numbers, from the distance to the true difference of Latitudes.

Probl. 7. *The Latitudes and difference of Longitude of two places being known, to sail by the great Circle from the one to the other.*



 In

In the Triangle *A, B, C,* let *A* represent *S. Christophers*, *C,* the *Lizard*, *B,* the North Pole, *A, B,* the Complement of the Latitude of *S. Christophers*, 74 *d.* 30 *m. B, C.* the Complement of the Latitude of the *Lizard*, 40 *d.* 0 *m.* and *A, B, C,* the difference of Longitude, 68 *d.* 30 *m.* Now therefore to steer a course from *A* to *C* alongst the Ark *A, C,* proceed thus:

1. By the 24 and 25 *Problems* of the fifth Chapter find the side *A, C,* as also the Angles *A,* and *C.*

2. By the 22 of the same Chapter find the Perpendicular *B, J,* cutting the side *A, C,* at Right Angles.

3. By the 8 of the same discover the Angle *A, B, I,* and by the 9 the side *A, I.*

4. Lessening the Angle *A, B, I,* two, five, or ten Degrees, as you shall see cause, (for *Example*, by the Angle *A, B, d,*) by the knowledge of the Angle *A, B, I,* and of the side *B, I,* find by the 11, 12, and 13 *Problems* of the same fifth Chapter, the Base *B, d,* the side *d. I,* and the Angle *B, d, I*; and so proceeding to do the like at the Points *e, f, g,* and *h,* you may thereby discover the several distances betwixt Point and Point, the several Latitudes at those Points, and the several Angles according to which you are to direct your Course: For at first, from *A* you are to steer according to the Angle *B, A, I,* until you shall have sailed so many Leagues as answer to the distance betwixt *A* and *d*: and then from *d,* according to the Angle *B, d, I,* untill you shall arrive at the Point *e,* according to the number of Leagues that *d* and *e* are distant the one from the other: and so consequently of the rest in their order, until you shall attain the Point *I,* from whence you are to steer full West towards *c,* the Angle *B, I, C*; being a Right Angle, *&c.*

CAP.

CAP. X.

The Use of the Rule of Proportion in the Gaging of Vessels.

Probl. 1. *The true Content of a Solid Measure being known, to find the Gage Point of the same Measure.*

THe Gage Point of a solid Measure is the Diameter of a Circle, whose Superficial Content is equal to the solid Content of the same Measure: so the solid Content of a Wine-gallon (according to *Winchester* measure) being 231 Cube-inches, if you conceive a Circle to contain so many Inches, you shall find (by the fortieth *Problem* of the fifth Chapter) the *Diameter* thereof to be 17. 15: For,

As 1 *is to* 1. 273 *: so is* 231 *to* 294. 1, *whose Square root* (by the twelfth *Problem* of the same Chapter) *is* 17. 15, *the Gage-point of Wine-measure.*

Thus likewise may you easily discover the Gage-point of Ale-measure, an Ale-gallon (as it hath been of late discovered) containing 288 Cube-inches: For,

As 1 *is to* 1. 273 *: so is* 288 *to* 366. 7, *whose Square-root is* 19. 15, *the Gage-point of Ale-measure.*

And (indeed) 288 Cube-inches ſeem to be the moſt probable Content of an Ale-gallon, being the ſixth part of 1728, which is the Number of Cube-inches contained in a Cube-foot. For ſo (according to that account) a Cube-foot contains juſt ſix Gallons; and the Gage-point of Ale-meaſure (by reaſon of the ſoil and waſte) exceeds that of Wine-meaſure juſt two Inches.

After the ſame manner alſo may you diſcover the Gage-point of any Forreign meaſure whatſoever, and afterwards by that means come to the knowledge of the true Content of their Veſſel, according to the Meaſures uſed amongſt them, as will plainly appear by that which ſhall hereafter be taught for the diſcovery of the Contents of Wine and Beer-veſſel according to the Engliſh Meaſures.

Now from that which is abovesaid doth neceſſarily follow this Corollary: *When the Diameter of a Cylinder in Inches is equal to the Gage-point of any Meaſure (given likewiſe in Inches) every Inch in the length thereof contains one Integer of the ſame Meaſure:* So in a Cylinder having 17.15 Inches Diameter, every Inch in the length thereof contains one intire Wine-gallon: and in another having 19.15 Inches Diameter, every Inch thereof contains one Ale-gallon, &c.

Probl. 2. *In a Wine or Beer-veſſel, the Diameters at the Head and Bongue being known, to find the required Diameter.*

Extend the Compaſſes upon the *Line of Inches* from the Diameter at the Head, to the Diameter at the Bongue: then applying that extent from the beginning of the ſame Line, and obſerving there the difference betwixt the two Diameters, (one of the Points remaining ſtill fixed at the beginning of the Line) cloſe the Compaſſes till the other Point may fall upon ſo many parts of the *Gage-line*, as the difference between the two Diameters, amounts unto in Inches: this done, and that extent applied from the Diameter

Diameter at the Head towards the Diameter at the Bongue, will cause the movable Point to fall upon the *Equated* Diameter you look for.

Example, The Diameter at the Head being 18. 3 Inches, and that at the Bongue 21. 5 Inches, I demand the *Equated* Diameter. First, extending the Compasses upon the Line of Inches from 18. 3 Inches to 21. 5, and then applying that extent from the beginning of the same Line, I find the movable Point to fall upon 3. 2 Inches, *viz.* the true difference of the two Diameters: Now therefore if still keeping one of the Points of the Compasses fixed at the beginning of that Line, I close them till the other Point may fall at 3. 2 upon the *Gage-line*, and after apply that extent from 18. 3 (the Diameter at the Head) the movable Point will at last fall upon 20. 54 Inches, the *Equated* Diameter required. And by this means your Vessel, which before was in part of an Oval form and irregular, is now reduced into a perfect *Cylinder*.

Probl. 2. *The equated Diameter and length of a Wine or Beer-vessel being given in Inches, to find the Content thereof in Wine-measure.*

The extent upon the *Line of Numbers* from 17. 15 (the Gage-point of Wine-measure) to the *Equated* Diameter, being twice repeated from the length, will reach to the Content in Wine-gallons.

Probl. 4. *To find the Content in Ale-measure.*

The extent from 19. 15 (the Gage-point of Ale-measure) to the *Equated* Diameter; being twice repeated from the length, will reach to the Content in Ale-gallons.

Probl. 5. *Having the length and the two Diameters at the Head and Bongue, together with the Equated Diameter and Content of a Vessel, and of which so much and no more of the liquor is drawn: that the Superficies thereof may cut some part of the Head, to find the true quantity of the remainder.*

Deduct

Deduct half the difference of the Diameters at t Head and Bongue, out of the diſtance intercep between the Bongue and the Superficies of the Liqu to the end you may thereby diſcover where the Li quor within the Veſſel cuts the Head, according which draw a Line with Chalke (or otherwiſe) up the Head, then having drawn another Line parallel the firſt, and of like diſtance from the other oppoſ ſide of the Head, you have in the middle of the He betwixt thoſe two Lines a Segment of the Veſſ marked out, and likewiſe two other Segments, t one above and the other below that middle Segmen after this taking the length of one of thoſe Parall in Inch-meaſure, the *Equated* Diameter of the Supe ficies may be thus found out upon the *Rule*:

The extent from the Diameter at the Head to the Equa ted Diameter of the Veſſel, will reach from the length one of the Parallels to the Equated Diameter of the Su perficies.

Then having diſcovered (by the *2d Problem* afor going) the *Equated* Diameter of thoſe two other E *quated* Diameters, find (by the tenth *Problem* of th fourth Chapter) the Mean Proportional between th third *Equated* Diameter and the diſtance between th two Parallels: This done, make uſe of that Mean Pro portional, as an *Equated* Diameter of the middle Seg ment, and then finding by one of the two laſt Pro *blems* according to the Queſtion propounded) the Con tent thereof in Gallons, *&c.* deduct that Content ou of the whole Content of the Veſſel: All this per formed, *when the Veſſel is above half full*, the Conten of that middle Segment and half that remainder be ing added together; is the Content you look for. Bu *when the veſſel is not half full*, half that remainder i the Content deſired.

C A P

CAP. XI.

The Use of the Rule of Proportion in Military Orders.

Probl. 1. *Any Number of Soldiers being propounded, to order them into a Square Battail of Men.*

FInd (by the twelfth *Problem* aforegoing (the Square-root of the Number given: For, look how much that Root shall happen to be, so many Soldiers ought you to place in Rank, and so many likewise in File, to make a Square Battail of Men.

Example; Let it be required to order 573 Soldiers into a Square Battail of Men: the Square-root of that Number is 23.94 : and therefore you are to place 23 in Rank, and as many also in File: For; Fractions are not considerable in Questions that concern, *Military* Orders.

Probl. 2. *Any Number of Souldiers being propounded to order them into a double Battail of Men: viz. which may have twice so many in Rank as in File.*

Find out the Square-root of half the Number given: for that Root is the Number of Soldiers to be placed in File: and so many more ought to be placed

placed in Rank, to make up a double Battail of Men.

Example, 1342 Souldiers being propounded to be put into that order: I find 26,&c. to be the Square-root of 671 (half the Number propounded) and thereupon conclude that 26 ought to be placed in File, and 52 in Rank, to order so many Soldiers into a double Battail of Men.

Probl. 3. *Any Number of Soldiers being given, to order them into a quadruple Battail: viz. such as may have four times so many in Rank as in File.*

Here the Square-root of the fourth part of the Number given will shew the Number to be placed in File, and sometimes so many are to be placed in Ranks.

So 2048 Soldiers being offered to be put into that order, 22 are to be placed in File, and 88 in Rank. For, the fourth part of 2048 is 512, whose Square-root is 22, &c.

Probl. 4. *Any Number of Soldiers being given, together with their distance in Rank and File, to order them into a Square Battail of Ground.*

Extend the Compasses upon the Mean Line of Numbers from the distance in File to the distance in Rank: this done, and that extent applied the same way, and upon the same Line from the Number of Soldiers propounded, will cause the movable Point to fall upon a fourth Number, whose Square-root appearing at the same Point upon the Great Line of Numbers is the whole Number of Men to be placed in File: by which if you divide the Number of Soldiers, the Quotient will shew the Number of Men to be placed in Rank.

Example, 2500 Men are propounded to be ordered into a Square Battail of Ground, in such sort that their distance in File being seven foot, and their distance in Rank three foot, the Ground whereupon they stand may be a just Square. To resolve this Question, extend the Compasses upon the Mean Line of

of Numbers downwards from 7 to 3: then (becauſe the fourth Number to be found in all likelihood will conſiſt of four Figures) if you apply that extent the ſame way from 2500 in the firſt part of the ſame Line, the movable Point will fall upon the fourth Number you look for, where alſo you may obſerve 32, &c. upon the ſecond part of the Great Line of Numbers, which are the Number of Men to be placed in File; again, if letting that Point of the Compaſſes remain fixed there, you cloſe them till the other Point may reach croſs-wiſe to 1 at the beginning of the firſt part of the ſaid Great Line of Numbers, that extent being applied the ſame way (*viz.* downwards and acroſs) from 2500 upon the ſame Great Line, the movable Point will fall near 76, &c. which are the Number of Soldiers to be placed in Rank.

Probl. 5. *Any Number of Soldiers being propounded, to order them in Rank and File according to the reaſon of any two Numbers given.*

This Problem is reſolved much after the ſame manner that the laſt was: For,

As the Proportional Number given for the File is to that given for the Rank: ſo is the Number of Souldiers to a fourth Number, whoſe Root is the Number of Men to be placed in Rank, by which if you divide the whole, the Quotient is the Number to be placed in File.

So if 2500 Soldiers were to be martialled in ſuch order, that the Number of Men to be placed in File might bear ſuch proportion to the Number of Men to be placed in Rank, as 5 bears to 12: I ſay then, as 5 is to 12, ſo is 2500 to another Number, whoſe Root is 77, &c. *viz.* the Number of Men to be placed in Rank, by which if the ſame 2500 be divided, the Quotient will be 32, &c. the Number of Men to be placed in File.

CAP. XII.

The Uſe of the Rule of Proportion in Queſtions that concern Intereſt and Annuities.

Probl. 1. *A Sum of Money being forborn for a certain time, to find how much it will be augmented at the expiration of the ſame time, accounting Intereſt upon Intereſt, according to a certain rate propounded.*

THe extent upon the Line of Numbers from 100 *l.* to the aggregate of 100 *l.* and the rate added together, being repeated the ſame way from the Sum given, ſo many times as there are years in the Queſtion, will at laſt cauſe the movable Point to fall upon the Principal increaſed with the Intereſt, according to the forbearance and rate propounded.

Example, I deſire to know how much 273 *l.* being forborn for five years will be increaſed at the expiration of thoſe years according to Intereſt upon

on Intereſt, and the rate of 8 *l. per centum:* Extend the Compaſſes upon the great Line of Numbers from 100 to 108: This done, if that extent be repeated five times from 273, the movable Point will at laſt fall upon 402. 1 (*viz.* 402 *l.* 2 *s.*) the Principal augmented with the Intereſt for the forbearance of thoſe five years.

Probl. 2. *A Sum of Money being due at a time to come, to find what it is worth in ready Money.*

This is the *Inverſe* of the laſt: for here, if you apply that extent backwards from the Number propounded, ſo many times as there are years in the Queſtion, you ſhall have your deſire.

Example, 402 *l.* 2 *s.* being due at the end of five years yet to come, I deſire to know how muoh that Sum is worth in ready Money according to the rate of 8 *l. per centum:* Extend the Compaſſes from 100 to 108, as before: And then, if you apply that extent five times downwards from 402. 1, the movable Point will at laſt fall upon 273 *l.* the value of 402. 1, in ready Money.

Probl. 3. *A yearly Rent or Annuity being forborn a certain Number of years, to find what the Arrearages thereof will amount unto according to any rate propounded.*

Firſt diſcover the principal that anſwers to the Rent or Annuity in queſtion, then find unto what Sum that Principal will be augmented (according to the given rate) at the end of the Term propounded: This done, if you ſubſtract the ſame Principal out of that Sum, the remainder is the Sum of the Arrearages you look for.

Example, A Rent or Annuity of 12 *l. per annum* being forborn 16 years, what will the Arrearages thereof amount unto, they being conceived to increaſe (as they grow due) after the rate of 8 *l. per centum.* Here firſt, to find the Principal that anſwers to 12 *l.* ſay thus: If 8 *l.* hath 100 l. for his Principal,

Principal, what ought 12 *l.* to have for his? the an-
swer will be (by the fourth *Problem* of the four
Chapter) 150 *l.* Having thus discovered the Princi-
pal of 12 *l.* *viz.* 150 *l.* I find (by the first *Problem*
of this Chapter) that the same 150 *l.* being forborn
16 years will amount (after the rate of 8 *l. per cen-
tum*) to 513. 9, that is 513 *l.* 18 *s.* Now therefore
if I deduct 150 *l.* (the Correspondent Principal to
the Annuity given) out of 513 *l.* 18 *s.* the remainder
viz. 363 *l.* 18 *s.* is the Sum of the Arearages requi-
red.

Probl. 4. *A yearly Rent or Annuity being propoun-
ded, to find what it is worth in ready money.*

First, find what the Arrearages thereof amount un-
to at the end of the Term propounded, and then
what those Arrearages are worth in ready money,
which shall likewise be the required price or value
of the Rent or Annuity propounded.

Example, What may a man which is desirous to
lay out his money after the rate of 8 *l. per centum*
afford to give for a Lease of 12 *l. per annum* that
hath yet 16 years in being? I find (by the last *Pro-
blem*) that the Arrearages of 12 *l. per annum*, being
forborn 16 years, amount then unto 363 *l.* 18 *s.* or
363. 9, and I find likewise (by the second *Problem*
aforegoing) that the same 363 *l.* 18 *s.* is worth in pre-
sent money 106. 2, or (which is all one) 106 *l.* 4 *s.* I
conclude therefore that the value of the Lease pro-
pounded (at the rate of 8 *l. per centum*) is 106 *l.* 4 *s.*

Here, when the Term of the Annuity begins not
presently, but after certain years to come, find what
the Arrearages forborn for all that time are worth in
ready money.

So in the *Example* last premised, if the Annuity of
16 years were not to begin till after the expiration
of 5 years, in this case you are to enquire what the
Arrearages (*viz.* 363 *l.* 18 *s.* being forborn 21 years
are worth in ready money, which you shall likewise
find

find (by the second *Problem* before cited) to be 72. 3, which being reduced is 72 *l.* 6 *s.* the value of the Lease required.

Probl. 5. *A Sum of Money being propounded, to find what Annuity (to continue any Number of Years, and according to any rate given) that Sum will buy.*

Take any Annuity at pleasure, then find the value of that Annuity in ready money: This done, the *Proportion* will be as followeth:

As the value found is to the Annuity taken; so is the Sum given to the Annuity required.

Example, What Annuity (to continue 16 Years) will 1205 *l.* deserve, so that the purchaser may gain after the rate of 8 *l. per centum?* Here, first, I take 12 *l. per annum* to continue 16 years, and find the value thereof in ready money (by the last *Problem*) to be 106. 2, or 106 *l.* 4 *s.* I say therefore,

If 106. 2 give 12 *l. per annum.*

What will 1205 *l.* yield? *Facit* 171,4 *per annum*, which being reduced is 171 *l.* 8 *s.* I conclude therefore, that 171 *l.* 8 *s.* is the Annuity (to endure 16 years) which 1205 *l.* doth deserve, after the rate of 8 *l. per centum.*

Deo Laus.

FINIS.

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